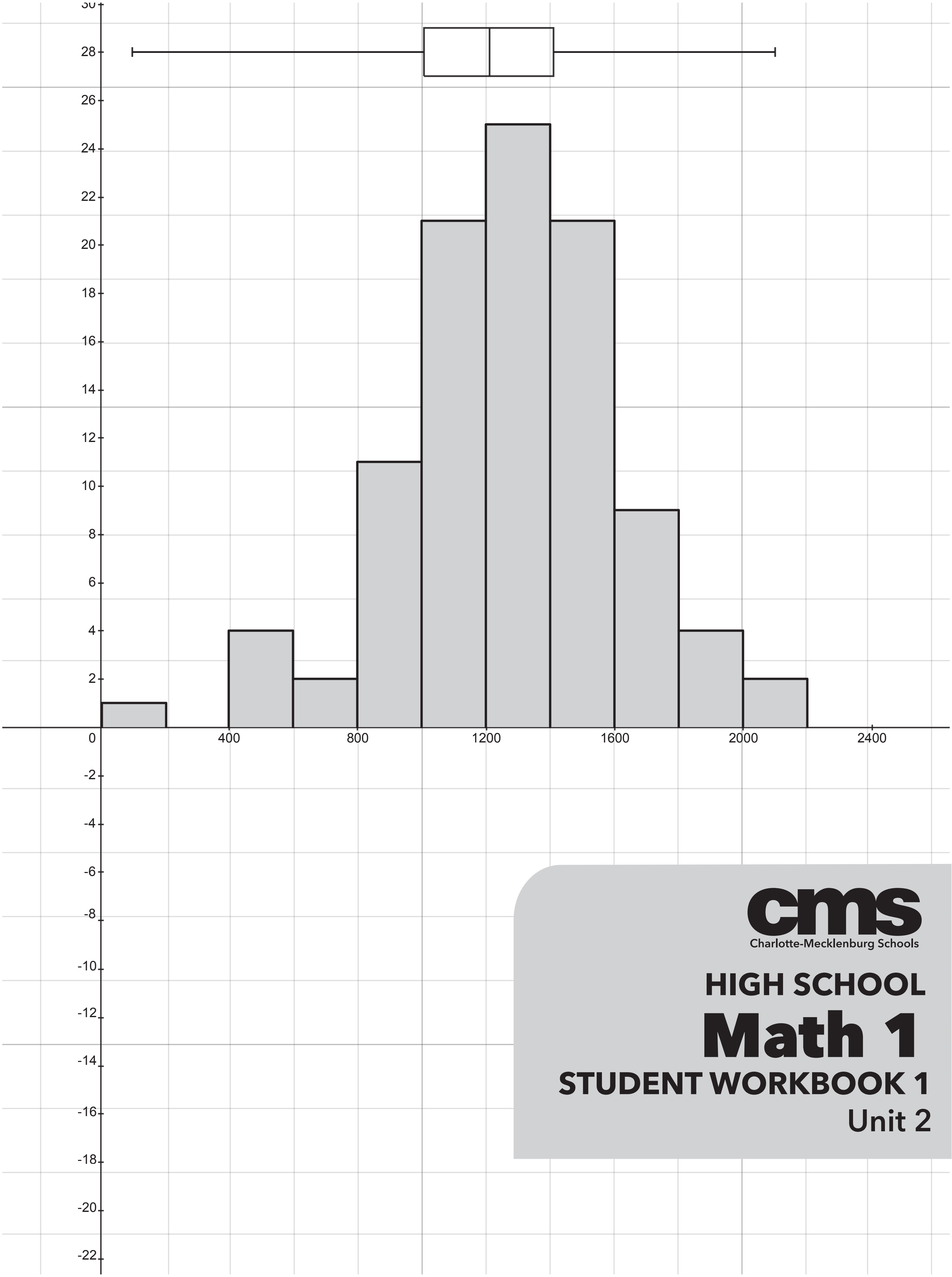


cms
Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

STUDENT WORKBOOK 1

Unit 2

Table of Contents

Lesson 1: Planning a Pizza Party

Learning Targets

- I can explain the meaning of the term "constraints."
- I can tell which quantities in a situation can vary and which ones cannot.
- I can use numbers and variables to write expressions representing the quantities in a situation.

Bridge

A zookeeper is preparing to care for snakes in an exhibit. For every feeding, she needs three mice and an additional two mice for each snake. How many mice are needed if the number of the snakes is:

a. 10

b. 6

c. x

Warm-up: A Main Dish and Some Side Dishes

Here are some variables and what they represent. All costs are in dollars.

- m represents the cost of a main dish.
- n represents the number of side dishes.
- s represents the cost of a side dish.
- t represents the total cost of a meal.

Equation	What does the equation mean in this situation?
a. $m = 7.50$	
b. $m = s + 4.50$	
c. $ns = 6$	
d. $m + ns = t$	

Activity 1: How Much Will It Cost?

Imagine your class is having a pizza party. Work with your group to plan what to order and to estimate what the party would cost.

1. Record your group's plan and cost estimate. What would it take to convince the class to go with your group's plan? Be prepared to explain your reasoning.

2. Write down one or more expressions that show how your group's cost estimate was calculated.

    a. In your expression(s), are there quantities that might change on the day of the party? Which ones?

    b. Rewrite your expression(s), replacing the quantities that might change with variables. Be sure to specify what the variables represent.

Are You Ready for More?

Find a pizza place near you and ask about the diameter and cost of at least two sizes of pizza. Compare the cost per square inch of the sizes.

Activity 2: What are the Constraints?

A constraint is something that limits what is possible or reasonable in a situation.

For example, one constraint in a pizza party might be the number of slices of pizza each person could have, s. We can write $s \le 3$ to say that each person gets three or fewer slices.

1. Look at the expressions you wrote when planning the pizza party earlier.
    a. Choose an expression that uses one or more variables.

    b. For each letter, determine what values would be reasonable. (For example, could the value be a non-whole number? A number greater than 50? A negative number? Exactly 2?)

2. Write equations or inequalities that represent some constraints in your pizza party plan. If a quantity must be an exact value, use the $=$ symbol. If it must be greater or less than a certain value to be reasonable, use the $<$, $>$, $\le$ or $\ge$ symbol.

Lesson Debrief

Lesson 1 Summary and Glossary

The expressions, equations, and inequalities you wrote in this lesson are mathematical **models**.

> **Model**: A mathematical representation used to describe quantities and their relationships in a real-life situation. It can be used to solve problems and make decisions.

Often, what we want to describe are **constraints**.

> **Constraint**: Something that limits what is possible or what is reasonable in a situation.

For example, when planning a birthday party, we might be dealing with these quantities and constraints:

quantities
- the number of guests
- the cost of food and drinks
- the cost of birthday cake
- the cost of entertainment
- the total cost

constraints
- 20 people maximum
- $5.50 per person
- $40 for a large cake
- $15 for music and $27 for games
- no more than $180 total cost

We can use both numbers and variables to represent the quantities. For instance, we can write 42 to represent the cost of entertainment, but we might use the letter n to represent the number of people at the party and the letter C for the total cost in dollars.

We can also write expressions using these numbers and variables. For instance, the expression $5.50n$ is a concise way to express the overall cost of food if it costs $5.50 per guest and there are n guests.

Sometimes a constraint is an exact value. For instance, the cost of music is $15. Other times, a constraint is a boundary or a limit. For instance, the total cost must be no more than $180. Symbols such as <, >,≤,≥ and = can help us express these constraints.

quantities

- the number of guests
- the cost of food and drinks
- the cost of birthday cake
- the cost of entertainment
- the total cost

constraints

- $n \le 20$
- $5.50n$
- 40
- $15 + 27$
- $C \le 180$

Equations can show the relationship between different quantities and constraints. For example, the total cost of the party is the sum of the costs of food, cake, entertainment. We can represent this relationship with:

$$C = 5.50n + 40 + 15 + 27 \qquad or \qquad C = 5.50n + 82$$

Deciding how to use numbers and variables to represent quantities, relationships, and constraints is an important part of mathematical modeling. Making assumptions—about the cost of food per person, for example—is also important in modeling.

A model such as $C = 5.50n + 82$ can be an efficient way to make estimates or predictions. When a quantity or a constraint changes, or when we want to know something else, we can adjust the model and perform a simple calculation, instead of repeating a series of calculations.

Unit 2 Lesson 1 Practice Problems

1. A balloon is released from a height of 5 feet and increases in height by 6.3 feet per second. Which expression represents the height of the balloon after x seconds?

   a. $5(6.3x)$

   b. $5+6.3x$

   c. $\frac{6.3x}{5}$

   d. $11.3x$

2. To support a local senior citizens center, a student club sent a flyer home to the n students in the school. The flyer said, "Please bring in money to support the senior citizens center. Paper money and coins accepted!" Their goal is to raise T dollars.

   Match each quantity to an expression, an equation, or an inequality that describes it.

Quantity	Expression, equation, or inequality
a. the dollar amount the club would have if they reached half of their goal b. the dollar amount the club would have if every student at the school donated 50 cents to the cause c. the dollar amount the club could donate if they made $50 more than their goal d. the dollar amount the club would still need to raise to reach its goal after every student at the school donated 50 cents e. the dollar amount the club would have if half of the students at the school each gave 50 cents	1. $T+50$ 2. $0.5T$ 3. $0.25n$ 4. $0.5n$ 5. $T-0.5n$

   a. __________

   b. __________

   c. __________

   d. __________

   e. __________

3. A student has scored a 76, 82, 80, 95 on the first four quizzes in math class. Write an expression that represents the average quiz score if the student scores x on the fifth quiz.

4. A student club started a fundraising effort to support animal rescue organizations. The club sent an information flyer home to the n students in the school. It says, "We welcome donations of any amount, including any change you could spare!" Their goal is to raise T dollars, and to donate to a cat shelter and a dog shelter.

   Match each quantity to an expression, an equation, or an inequality that describes it.

Quantity	Expression, equation, or inequality
a. the dollar amount the club would still need to raise to reach its goal after every student at the school donated a quarter	1. $\frac{3}{4}n \cdot \frac{1}{2}$
b. the dollar amount the club would have if every student at the school donated a quarter to the cause	2. $\frac{1}{4}T$
c. the dollar amount the club would have if three-fourths of the students at the school each gave 50 cents	3. $T - \frac{1}{4}n$
d. the dollar amount the club could donate to the cat shelter if they reached their goal and gave a quarter of the total donation to a dog shelter	4. $\frac{3}{4}T$
e. the dollar amount the club would have if they reached one-fourth of their goal	5. $\frac{1}{4}n$

a. __________

b. __________

c. __________

d. __________

e. __________

5. A softball team is ordering pizza to eat after their tournament. They plan to order cheese pizzas that cost $6 each and four-topping pizzas that cost $10 each. They order c cheese pizzas and f four-topping pizzas. Which expression represents the total cost of all of the pizzas they order?

   a. $6 + 10$

   b. $c + f$

   c. $6c + 10f$

   d. $6f + 10c$

6. The values of coins in the pockets of several students are recorded. Find the mean of the values: 10, 20, 35, 35, 35, 40, 45, 45, 50, 60

   a. 10 cents

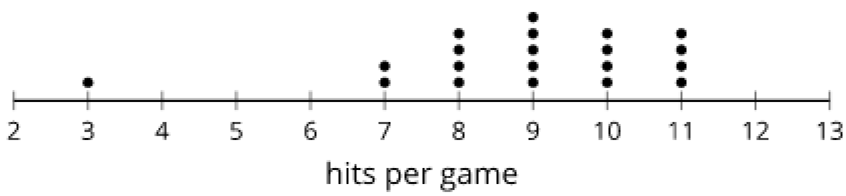
   b. 35 cents

   c. 37.5 cents

   d. 50 cents

(From Unit 1)

7. The dot plot displays the number of hits a baseball team made in several games. The distribution is skewed to the left.



If the game with 3 hits is considered to be recorded in error, it might be removed from the data set. If that happens:

   a. What happens to the mean of the data set?

   b. What happens to the median of the data set?

(From Unit 1)

8. A set of data has a standard deviation of 0, and one of the data values is 14. What can you say about the data values?

(From Unit 1)

9. A zookeeper is preparing to care for snakes in an exhibit. For each question, write an expression representing the supplies needed.

   a. The zookeeper needs 4.5 ounces of crickets for each snake. How many ounces of crickets are needed if the number of snakes is:

      - 10
      - 6
      - x

   b. There is one male snake, and the rest are female. The zookeeper needs one vitamin pill for every female snake. How many vitamin pills does she need if the number of snakes is:

      - 10
      - 6
      - x

(Addressing NC.6.EE.6)

Lesson 2: Writing Equations to Model Relationships (Part One)

Learning Targets

- I can tell which quantities in a situation can vary and which ones cannot.
- I can use numbers and variables to write equations representing the relationships in a situation.

Bridge

For each problem, identify any important quantities. If it's a known quantity, write the number and a short description of what it represents. If it's an unknown quantity, assign a variable to represent it and write a short description of what that variable represents.

1. Elena is going to mow lawns for the summer to make some extra money. She will charge $20 for every lawn she mows and plans on mowing several lawns each week.

2. Tyler is packing his bags for vacation. He plans to pack two outfits for each day of vacation.

Warm-up: Percent of 200

Evaluate mentally.

- 25% of 200
- 12% of 200
- 8% of 200
- p% of 200

Activity 1: Blueberries and Earnings

1. Three friends visit a local Farmer's Market and purchase blueberries. Write an equation to represent each situation.
   a. Blueberries are \$4.99 per pound. Paul buys $\frac{3}{4}$ pound of blueberries and pays r dollars.

   b. Blueberries are \$4.99 per pound. Diego buys b pounds of blueberries and pays \$14.95.

   c. Blueberries are \$4.99 per pound. Jada buys p pounds of blueberries and pays c dollars.

   d. Blueberries are d dollars per pound. Lin buys q pounds of blueberries and pays t dollars.

2. Noah and Mai have summer jobs. Write an equation to represent each situation.
   a. Noah earned \$400 over the summer. Mai earned p dollars, which is \$45 more than Noah did.

   b. Noah earned n dollars over the summer. Mai earned \$275, which is \$45 more than Noah did.

   c. Noah earned v dollars over the summer. Mai earned m dollars, which is 45 dollars more than Noah did.

   d. Noah earned w dollars over the summer. Mai earned x dollars, which is y dollars more than Noah did.

3. How are the equations you wrote for the blueberry purchases like the equations you wrote for Mai and Noah's summer earnings? How are they different?

Activity 2: Car Prices

The tax on the sale of a car in North Carolina is 3%. At a dealership in Concord, a car purchase also involves $120 in miscellaneous charges.

There are several quantities in this situation: the original car price, sales tax, miscellaneous charges, and total price. Write an equation to describe the relationship between all the quantities when:

a. The original car price is $9,500.

b. The original car price is $14,699.

c. The total price is $22,480.

d. The original price is p.

Are You Ready for More?

How would each equation you wrote change if the tax on car sales is r% and the miscellaneous charges are m dollars?

Lesson Debrief

Lesson 2 Summary and Glossary

Suppose your class is planning a trip to a museum. The cost of admission is $7 per person and the cost of renting a bus for the day is $180.

- If 24 students and 3 teachers are going, we know the cost will be: $7(24) + 7(3) + 180$ or $7(24 + 3) + 180$.
- If 30 students and 4 teachers are going, the cost will be: $7(30 + 4) + 180$.

Notice that the numbers of students and teachers can vary. This means the cost of admission and the total cost of the trip can also vary, because they depend on how many people are going.

Variables are helpful for representing quantities that vary. If s represents the number of students who are going, t represents the number of teachers, and C represents the total cost, we can model the quantities and constraints by writing:

$$C = 7(s + t) + 180$$

Some quantities may be fixed. In this example, the bus rental costs $180 regardless of how many students and teachers are going (assuming only one bus is needed).

Variables can also be used to represent quantities that are constant. We might do this when we don't know what the value is, or when we want to understand the relationship between quantities (rather than the specific values).

For instance, if the bus rental is B dollars, we can express the total cost of the trip as $C = 7(s + t) + B$. No matter how many teachers or students are going on the trip, B dollars need to be added to the cost of admission.

Unit 2 Lesson 2 Practice Problems

1. Large cheese pizzas cost \$5 each, and large one-topping pizzas cost \$6 each.

   Write an equation that represents the total cost, T, of c large cheese pizzas and d large one-topping pizzas.

2. Jada plans to serve milk and healthy cookies for a book club meeting. She is preparing 12 ounces of milk and 4 cookies per person. Including herself, there are 15 people in the club. A package of cookies contains 24 cookies and costs \$4.50.

   A 1-gallon jug of milk contains 128 ounces and costs \$3. Let n represent number of people in the club, m represent the ounces of milk, c represent the number of cookies, and b represent Jada's budget in dollars.

   Select **all** of the equations that could represent the quantities and constraints in this situation.

   a. $m = 12(15)$

   b. $3m + 4.5c = b$

   c. $4n = c$

   d. $4(4.50) = c$

   e. $b = 2(3) + 3(4.50)$

3. A student on the track team runs 45 minutes each day as a part of her training. She begins her workout by running at a constant rate of 8 miles per hour for a minutes, then slows to a constant rate of 7.5 miles per hour for b minutes.

   Which equation describes the relationship between the distance she runs in miles, D, and her running speed, in miles per hour?

   a. $a + b = 45$

   b. $8a + 7.5b = D$

   c. $8(\frac{a}{60}) + 7.5(\frac{b}{60}) = D$

   d. $8(45 - b) + 7.5b = D$

4. Elena bikes 20 minutes each day for exercise.

   Write an equation to describe the relationship between her distance in miles, D, and her biking speed, in miles per hour, when she bikes:

   a. at a constant speed of 13 miles per hour for the entire 20 minutes

   b. at a constant speed of 15 miles per hour for the first 5 minutes, then at 12 miles per hour for the last 15 minutes

   c. at a constant speed of M miles per hour for the first 5 minutes, then at N miles per hour for the last 15 minutes

5. A coach for a little league baseball team is ordering trophies for the team. Players on the team are allowed to choose between two types of trophies. The gold baseball trophies cost $5.99 each, and the uniform baseball trophies cost $6.49 each. The team orders g gold baseball trophies and u uniform baseball trophies.

   Write an expression that could represent the total cost of all of the trophies.

   (From Unit 2, Lesson 1)

6. In a trivia contest, players form teams and work together to earn as many points as possible for their team. Each team can have between three and five players. Each player can score up to 10 points in each round of the game. Elena and four of her friends decided to form a team and play a round.

   Write an expression, an equation, or an inequality for each quantity described here. If you use a variable, specify what it represents.

   a. the number of points that Elena's team earns in one round

   b. the number of points Elena's team earns in one round if every player scores between six and eight points

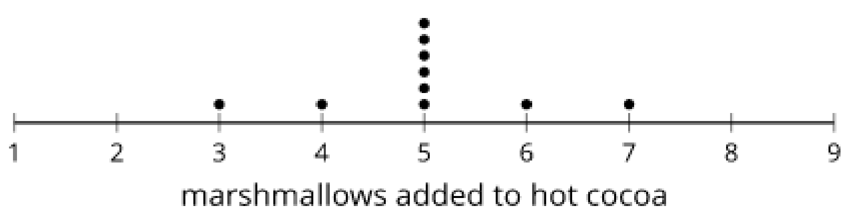
   c. the number of points Elena's team earns if each player misses one point

   d. the number of players in a game if there are five teams of four players each

   e. the number of players in a game if there are at least three teams

   (From Unit 2, Lesson 1)

7. The dot plot displays the number of marshmallows added to hot cocoa by several kids. Which is the standard deviation of the data represented in the dot plot?



   a. 0.2 marshmallow

   b. 1 marshmallow

   c. 2 marshmallows

   d. 5 marshmallows

   (From Unit 1)

8. Here is a data set from a research study:

5	10	10	15	100

a. After studying the data, the researcher realized that the value 100 was meant to be recorded as 15. What happens to the mean and standard deviation of the data set when the 100 is changed to a 15?

b. For the original data set, with the 100, would the median or the mean be a better choice of measure for the center? Explain your reasoning.

(From Unit 1)

9. For each problem, identify any important quantities. If it's a known quantity, write the number and a short description of what it represents. If it's an unknown quantity, assign a variable to represent it and write a short description of what that variable represents.

a. Clare is in charge of getting snacks for a road trip with her friends and her dog. She has $35 to go to the store to get some supplies. The snacks for herself and her friends cost $3.25 each, and her dog's snacks cost $9 each.

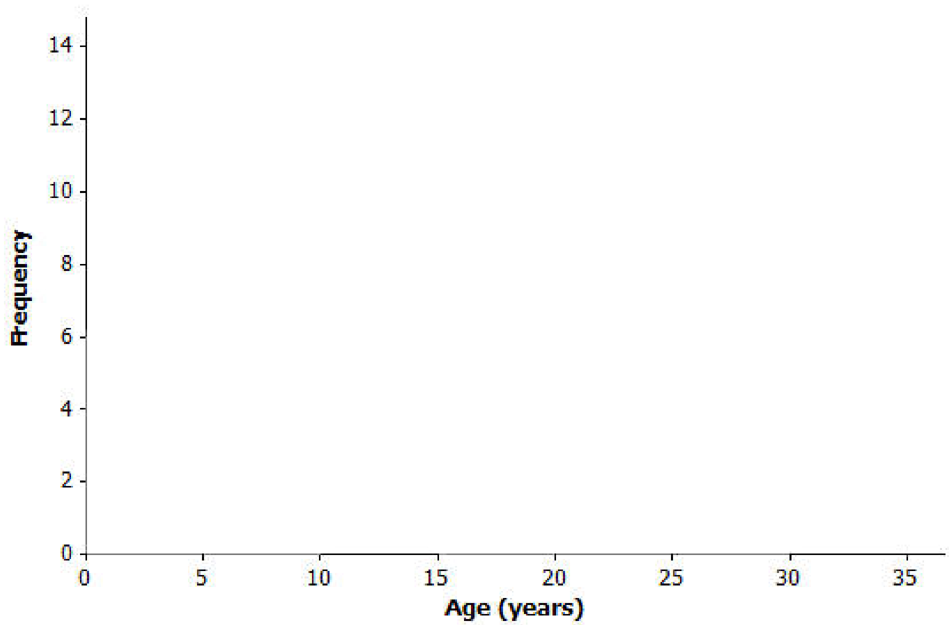
b. Mai's teacher orders tickets to the local carnival for herself, the entire class, and three more chaperones. Student tickets are $4.50.

(Addressing NC.6.EE.6)

10. Twenty-five people were attending an event. The ages of the people are as follows[1]:

3, 3, 4, 4, 4, 4, 5, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 16, 17, 22, 22, 25.

a. Create a histogram of the ages using the provided axes.



b. Would you describe your graph as symmetrical or skewed? Explain your choice.

c. Identify a typical age of the 25 people.

d. What event do you think the 25 people were attending? Use your histogram to justify your conjecture.

(From Unit 1)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Lesson 3: Writing Equations to Model Relationships (Part Two)

Learning Targets

- I can use words and equations to describe the patterns I see in a table of values or in a set of calculations.
- When given a description of a situation, I can use representations like diagrams and tables to help make sense of the situation and write equations for it.

Bridge

Complete the table so that each pair of numbers makes the equation true.

$$y = 3x$$

x	y
-5	
	96
$\frac{2}{3}$	

Warm-up: Finding a Relationship

Here is a table of values where the two quantities x and y are related.

What are some strategies you could use to find a relationship between x and y? Brainstorm as many ways as possible.

x	y
1	0
3	8
5	24
7	48

Activity 1: What Are the Relationships?

1. Each table below represents the relationship between two quantities.

Table A

Meters from home, x	0	75	128	319	396
Meters from school, y	400	325	272	81	4

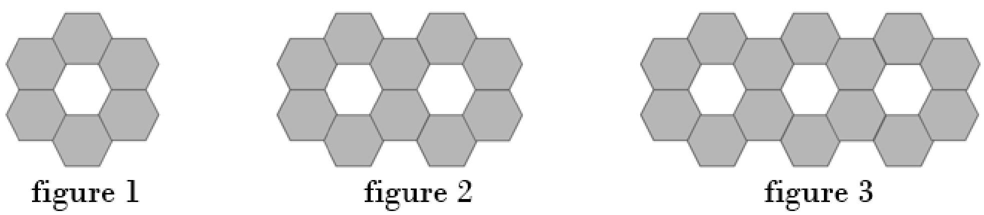
Table B

Electricity bills in dollars, x	85	124	309	816
Total expenses in dollars, y	485	524	709	1,216

   a. Describe in words how the two quantities in each table are related.

   b. Write an equation to represent the relationship between the two quantities.

2. Each figure below is created using hexagons.



   a. What is the relationship between the figure number and the number of hexagons in the figure?

   b. Write an equation to represent the relationship between the figure number and the number of hexagons.

   c. Use your equation to determine the number of hexagons in figure 100.

3. A $\frac{1}{2}$-gallon jug of milk can fill 8 cups, while 32 fluid ounces of milk can fill 4 cups.

   a. What is the relationship between the number of gallons and ounces?

   b. Write an equation to represent the relationship between the two quantities.

Are You Ready for More?

Each figure to the right is created using stars.

What is the relationship between the figure number and the number of stars in the figure? Represent the relationship in as many ways as possible.

Lesson Debrief

Lesson 3 Summary and Glossary

Sometimes, the relationship between two quantities is easy to see. For instance, we know that the perimeter of a square is always 4 times the side length of the square. If P represents the perimeter and s the side length, then the relationship between the two measurements (in the same unit) can be expressed as $P = 4s$, or $s = \frac{P}{4}$.

Other times, the relationship between quantities might take a bit of work to figure out—by doing calculations several times or by looking for a pattern. Here are two examples.

- A plane departed from New Orleans and is heading to San Diego. The table shows its distance from New Orleans, x, and its distance from San Diego, y, at some points along the way.

Miles from New Orleans	Miles from San Diego
100	1,500
300	1,300
500	1,100
	1,020
900	700
1,450	
x	y

What is the relationship between the two distances? Do you see any patterns in how each quantity is changing? Can you find out what the missing values are?

Notice that every time the distance from New Orleans increases by some number of miles, the distance from San Diego decreases by the same number of miles, and that the sum of the two values is always $1,600$ miles.

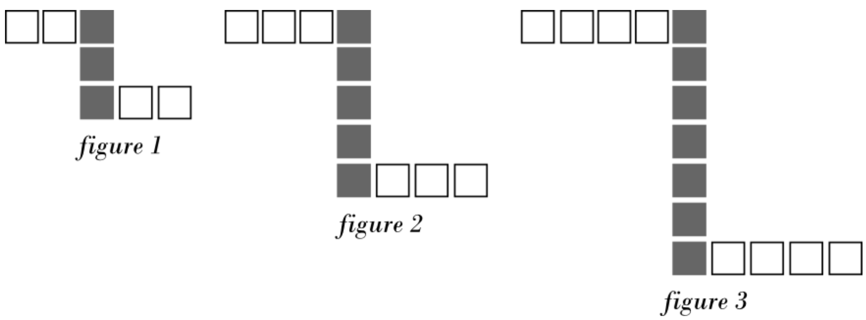
The relationship can be expressed with any of these equations:

$$x + y = 1,600$$

$$y = 1,600 - x$$

$$x = 1,600 - y$$

- In the following pattern, the number of squares is related to the figure number.



There are a variety of ways to look for a pattern and express the relationship. One way is with a table.

Figure number	Number of squares
1	7
2	11
3	15
4	
5	

Do you notice a pattern?

- As the figure number increases by 1, the number of squares increases by 4.
- $4(1) + 3 = 7$ and $4(2) + 3 = 11$ and $4(3) + 3 = 15$

We can generalize that the number of squares is 4 times the figure number plus 3. This can be expressed by the equation $s = 4f + 3$ where f is the figure number and s is the number of squares.

Unit 2 Lesson 3 Practice Problems

1. A landscaping company is delivering crushed stone to a construction site. The table shows the total weight in pounds, W, of n loads of crushed stone.

   Which equation could represent the total weight, in pounds, for n loads of crushed stone?

Number of loads of crushed stone	Total weight in pounds
0	0
1	$2,000$
2	$4,000$
3	$6,000$

   a. $W = \frac{6,000}{n}$

   b. $W = 6,000 - 2,000n$

   c. $W = 2,000n$

   d. $W = n + 2,000$

2. Tyler needs to complete this table for his consumer science class. He knows that 1 tablespoon contains 3 teaspoons and that 1 cup contains 16 tablespoons.

Number of teaspoons	Number of tablespoons	Number of cups
		2
36	12	
	48	3

   a. Complete the missing values in the table.

   b. Write an equation that represents the number of teaspoons, t, contained in a cup, C.

3. The volume of dry goods, like apples or peaches, can be measured usings bushels, pecks, and quarts. A bushel contains 4 pecks, and a peck contains 8 quarts.

   What is the relationship between number of bushels, b, and the number of quarts, q? If you get stuck, try creating a table.

4. Elena has $225 in her bank account. She takes out $20 each week for w weeks. After w weeks she has d dollars left in her bank account.

   Write an equation that represents the amount of money left in her bank account after w weeks.

   (From Unit 2, Lesson 2)

5. Priya is hosting a poetry club meeting this week and plans to have fruit punch and cheese for the meeting. She is preparing 8 ounces of fruit punch per person and 2 ounces of cheese per person. Including herself, there are 12 people in the club.

   A package of cheese contains 16 ounces and costs $3.99. A one-gallon jug of fruit punch contains 128 ounces and costs $2.50. Let p represent number of people in the club, f represent the ounces of fruit punch, c represent the ounces of cheese, and b represent Priya's budget in dollars.

   Select **all** of the equations that could represent the quantities and constraints in this situation.

   a. $f = 8 \cdot 12$

   b. $c = 2 \cdot 3.99$

   c. $2 \cdot 3.99 + 2.50 = b$

   d. $2p = c$

   e. $8f + 2c = b$

   (From Unit 2, Lesson 2)

6. The data show the number of free throws attempted by a team in its first 10 games.

2	11	11	11	12	12	13	14	14	15

The median is 12 attempts, and the mean is 11.5 attempts. After reviewing the data, it is determined that 2 should not be included, since that was an exhibition game rather than a regular game during the season.

a. What happens to the median if the 2 (attempts in first game) is removed from the data set?

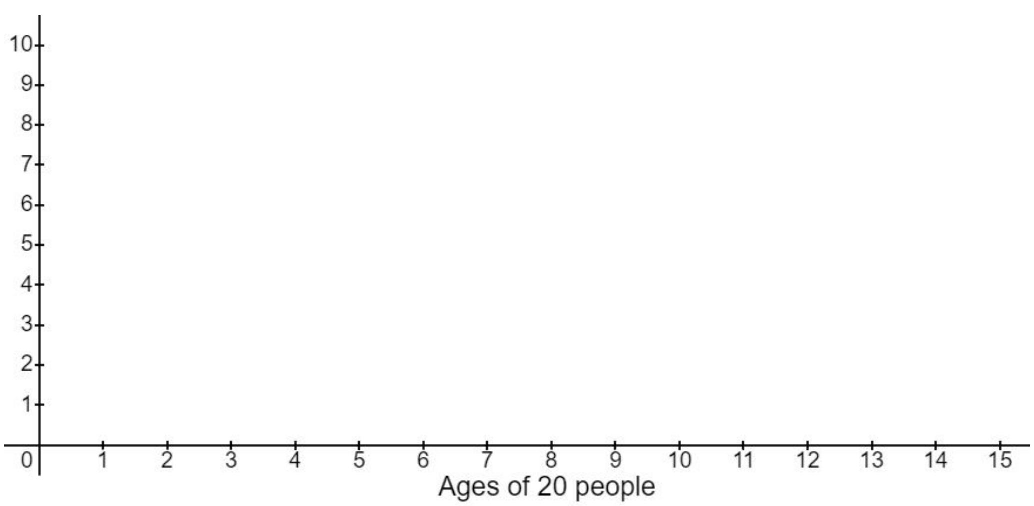
b. What happens to the mean if the 2 (attempts in first game) is removed from the data set?

(From Unit 1)

7. The standard deviation for a data set is 0. What can you conclude about the data?

(From Unit 1)

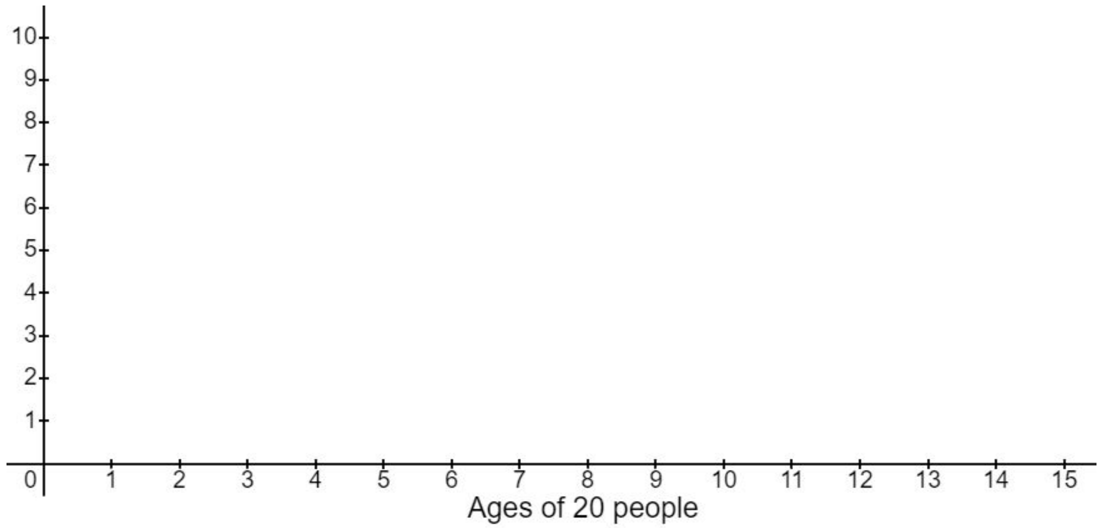
8. Draw a histogram of a data distribution representing the ages of 20 people for which the median and the mean would be approximately the same.[1]



(From Unit 1)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

9. Draw a histogram of a data distribution representing the ages of 20 people for which the median is noticeably less than the mean.[2]



(From Unit 1)

10. Complete the table so that each pair of numbers makes the equation true.

a. $m = 2n + 1$

n	m
3	
	5
	12

b. $d = \frac{16}{e}$

e	d
4	
-3	
	2

(Addressing NC.6.EE.9)

[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

Lesson 4: Equations and Their Solutions

Learning Targets

- I can explain what it means for a value or pair of values to be a solution to an equation.
- I can find solutions to equations by reasoning about a situation or by using algebra.

Bridge

Some students are trying to write an expression with fewer terms that is equivalent to $8-3(4-9x)$. For each student, circle if you agree or disagree and share your reasoning, including finding and describing any errors that may have occurred.[1]

Noah says, "I worked the problem from left to right and ended up with $20-45x$."	Lin says, "I started inside the parentheses and ended up with $23x$."	Jada says, "I used the distributive property and ended up with $-4+27x$."	Andre says, "I also used the distributive property, but I ended up with $-4-27x$."
$8-3(4-9x)$ $5(4-9x)$ $20-45x$	$8-3(4-9x)$ $8-3(-5x)$ $8+15x$ $23x$	$8-3(4-9x)$ $8-(12-27x)$ $8-12-(-27x)$ $-4+27x$	$8-3(4-9x)$ $8-12-27x$ $-4-27x$
Agree Disagree	Agree Disagree	Agree Disagree	Agree Disagree

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Warm-up: What Is a Solution?

A granola bite contains 27 calories. Most of the calories come from c grams of carbohydrates. The rest come from other ingredients. One gram of carbohydrate contains 4 calories.

The equation $4c + 5 = 27$ represents the relationship between these quantities.

1. What could the 5 represent in this situation?

2. Priya said that neither 8 nor 3 could be the solution to the equation. Explain why she is correct.

3. Find the solution to the equation.

Activity 1: Calories from Protein and Fat

One gram of protein contains 4 calories. One gram of fat contains 9 calories. A snack has 60 calories from p grams of protein and f grams of fat.

The equation $4p + 9f = 60$ represents the relationship between these quantities.

Identify pairs of values that could be the number of grams of protein and number of grams of fat in the snack. Record the pairs you try in a table like the one shown below. Be prepared to explain your reasoning.

Number of grams of protein	Number of grams of fat	Is this a possible pair of values for the 60 calorie snack? Yes or No

Lesson Debrief

Lesson 4 Summary and Glossary

An equation that contains only one unknown quantity or one quantity that can vary is called an "equation in one variable."

For example, the equation $8h + 50 = 250$ represents the fact that Clare began with $\$50$ saved, worked for h hours at a job that paid $\$8$ per hour, and now has $\$250$.

This is an equation in one variable, because h is the only quantity that we don't know. To solve this equation means to find a value of h that makes the equation true.

In this case, 25 is the solution because substituting 25 for h in the equation results in a true statement.

$$\begin{aligned} 8h + 50 &= 250 \\ 8(25) + 50 &= 250 \\ 200 + 50 &= 250 \\ 250 &= 250 \end{aligned}$$

An equation that contains two unknown quantities or two quantities that vary is called "an equation in two variables." A solution to such an equation is *a* pair of numbers that makes the equation true.

Suppose Tyler spends $\$45$ on T-shirts and socks. A T-shirt costs $\$10$, and a pair of socks costs $\$2.50$. If t represents the number of T-shirts and p represents the number of pairs of socks that Tyler buys, we can represent this situation with the equation:

$$10t + 2.50p = 45$$

This is an equation in two variables. More than one pair of values for t and p make the equation true.

$t = 3$ and $p = 6$

$$\begin{aligned} 10(3) + 2.50(6) &= 45 \\ 30 + 15 &= 45 \\ 45 &= 45 \end{aligned}$$

$t = 4$ and $p = 2$

$$\begin{aligned} 10(4) + 2.50(2) &= 45 \\ 40 + 5 &= 45 \\ 45 &= 45 \end{aligned}$$

$t = 2$ and $p = 10$

$$\begin{aligned} 10(2) + 2.50(10) &= 45 \\ 20 + 25 &= 45 \\ 45 &= 45 \end{aligned}$$

In this situation, one constraint is that the combined cost of shirts and socks must equal $\$45$. Solutions to the equation are pairs of t and p values that satisfy this constraint.

Combinations such as $t = 1$ and $p = 10$ or $t = 2$ and $p = 7$ are *not* solutions because they don't meet the constraint. When these pairs of values are substituted into the equation, they result in statements that are false.

Unit 2 Lesson 4 Practice Problems

1. An artist is selling children's crafts. Necklaces cost \$2.25 each, and bracelets cost \$1.50 each. Select **all** the combinations of necklaces and bracelets that the artist could sell for exactly \$12.00.

    a. 5 necklaces and 1 bracelet

    b. 2 necklaces and 5 bracelets

    c. 3 necklaces and 3 bracelets

    d. 4 necklaces and 2 bracelets

    e. 3 necklaces and 5 bracelets

    f. 6 necklaces and no bracelets

    g. No necklaces and 8 bracelets

2. Diego is collecting dimes and nickels in a jar. He has collected \$22.25 so far. The relationship between the numbers of dimes and nickels, and the amount of money in dollars, is represented by the equation $0.10d + 0.05n = 22.25$.

    Select **all** the values (d, n) that could be solutions to the equation.

    a. $(0, 445)$

    b. $(0.50, 435)$

    c. $(233, 21)$

    d. $(118, 209)$

    e. $(172, 101)$

3. Volunteer drivers are needed to bring 80 students to the championship baseball game. Drivers either have cars, which can seat 4 students, or vans, which can seat 6 students. The equation $4c + 6v = 80$ describes the relationship between the number of cars, c, and number of vans, v, that can transport exactly 80 students.

    Select **all** statements that are true about the situation.

    a. If 12 cars go, then 2 vans are needed.

    b. $c = 14$ and $v = 4$ are a pair of solutions to the equation.

    c. If 6 cars go and 11 vans go, there will be extra space.

    d. 10 cars and 8 vans aren't enough to transport all the students.

    e. If 20 cars go, no vans are needed.

    f. 8 vans and 8 cars are numbers that meet the constraints in this situation.

4. The drama club is printing T-shirts for its members. The printing company charges a certain amount for each shirt plus a setup fee of $40. There are 21 students in the drama club.

   a. If there are 21 students in the club and the t-shirt order costs a total of $187, how much does each t-shirt cost? Show your reasoning.

   b. The equation $201.50 = f + 6.50(21)$ represents the cost of printing the shirts at a second printing company. Find the solution to the equation and state what it represents in this situation.

5. Identify the error in generating an expression equivalent to $4 + 2x - \frac{1}{2}(10 - 4x)$. Then correct the error.[2]

$$4 + 2x + -\tfrac{1}{2}(10 + -4x)$$
$$4 + 2x + -5 + 2x$$
$$4 + 2x - 5 + 2x$$
$$-1$$

6. Kiran's family is having people over to watch a football game. They plan to serve sparkling water and pretzels. They are preparing 12 ounces of sparkling water and 3 ounces of pretzels per person. Including Kiran's family, there will be 10 people at the gathering. A bottle of sparkling water contains 22 ounces and costs $1.50. A package of pretzels contains 16 ounces and costs $2.99. Let n represent number of people watching the football game, s represent the ounces of sparkling water, p represent the ounces of pretzels, and b represent Kiran's budget in dollars.

   Which equation best represents Kiran's budget?

   a. $12s + 3p = b$

   b. $12 \cdot 10 + 3 \cdot 10 = b$

   c. $1.50s + 2.99p = b$

   d. $1.50 \cdot 6 + 2.99 \cdot 2 = b$

   (From Unit 2, Lesson 2)

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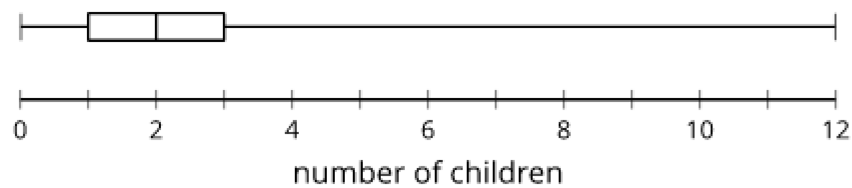
7. The speed of an object can be found by taking the distance it travels and dividing it by the time it takes to travel that distance. An object travels 100 feet in 2.5 seconds. Let the speed, S, be measured in feet per second.

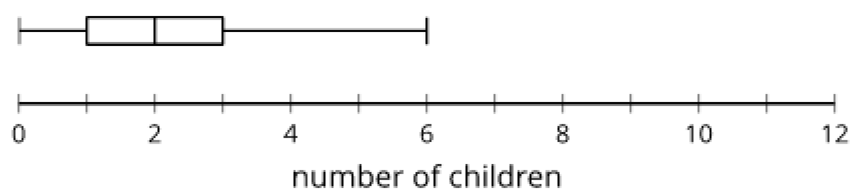
   Write an equation to represent the relationship between the three quantities (speed, distance, and time).

   (From Unit 2, Lesson 2)

8. The box plot represents the distribution of the number of children in 30 different families.



   After further examination, the value of 12 is removed for having been recorded in error. The box plot represents the distribution of the same data set, but with the maximum, 12, removed.



   The median is 2 children for both plots.

   a. Explain why the median remains the same when 12 was removed from the data set.

   b. When 12 is removed from the data set, does the mean remain the same? Explain your reasoning.

   (From Unit 1)

9. The number of points Jada's basketball team scored in their games has a mean of about 44 and a standard deviation of about 15.7 points.

   Interpret the mean and standard deviation in the context of Jada's basketball team.

   (From Unit 1)

10. Noah says that $9x - 2x + 4x$ is equivalent to $3x$ because the subtraction sign tells us to subtract everything that comes after $9x$. Elena says that $9x - 2x + 4x$ is equivalent to $11x$ because the subtraction only applies to $2x$.

    Do you agree with either of them? Explain your reasoning.[3]

    (Addressing NC.7.EE.1; NC.7.EE.3)

Lesson 5: Equivalent Equations

Learning Targets

- I can tell whether two expressions are equivalent and explain why or why not.
- I know and can identify the moves that can be made to transform an equation into an equivalent one.
- I understand what it means for two equations to be equivalent, and how equivalent equations can be used to describe the same situation in different ways.

Warm-up: Two Expressions

Your teacher will assign you one of these expressions:

$$\frac{n^2-9}{2(4-3)} \quad \textit{or} \quad (n+3)\cdot\frac{(n-3)}{8-3\cdot 2}$$

Evaluate your expression when n is:

1. 5
2. 7
3. 13
4. -1

Activity 1: What's Acceptable?

1. Are any of these equations equivalent to one another? If so, which ones? Explain your reasoning.

   $3a + 6 = 15$

   $3a = 9$

   $a + 2 = 5$

   $\frac{1}{3}a = 1$

Noah is buying a pair of jeans and using a coupon for 10% off. The total price is \$56.70, which includes \$2.70 in sales tax.

Noah's purchase can be modeled by the equation:

$$x - 0.1x + 2.70 = 56.70$$

2. Discuss with a partner:
   a. What does the solution to the equation mean in this situation?
   b. How can you verify that 70 is not a solution, but 60 is the solution?

3. Your teacher will give you a set of six cards. Three of the cards have equations that are related to $x - 0.1x + 2.70 = 56.70$. The other three cards have interpretations in terms of Noah's purchase. Match the cards together.

4. The table below shows the three equations from the card sort. Each equation is a result of performing one or more moves on the original equation describing Noah's purchase. For each equation:

   a. What move was made?

   b. Check if 60 is the solution to the equation.

Original Equation: $x - 0.1x + 2.70 = 56.70$			
Equation	**Card #**	**What was done to the original equation to get this equation?**	**Is 60 the solution to the equation? (yes or no)**
$100x - 10x + 270 = 5{,}670$			
$x - 0.1x = 54$			
$0.9x + 2.70 = 56.70$			

5. Andre said he found two more equations that are equivalent to the original. Mai says they aren't equivalent because 60 isn't the solution to the equations. Is Mai correct? If so, what moves did Andre make that weren't "acceptable"?

   $2(x - 0.1x + 2.70) = 56.70$ $\qquad$ $x - 0.1x = 59.40$

Are You Ready for More?

Here is a puzzle:

$$\begin{aligned} m+m &= N \\ N+N &= p \\ m+p &= Q \\ p+Q &= ? \end{aligned}$$

Which expressions could be equal to $p+Q$?

a. $2p+m$

b. $4m+N$

c. $3N$

d. $9m$

Lesson Debrief

Lesson 5 Summary and Glossary

Suppose we bought two packs of markers and a $\$0.50$ glue stick for $\$6.10$. If m is the dollar cost of one pack of markers, the equation $2m+0.50=6.10$ represents this purchase. The solution to this equation is 2.80.

Now suppose a friend bought six of the same packs of markers and three $\$0.50$ glue sticks, and paid $\$18.30$. The equation $6m+1.50=18.30$ represents this purchase. The solution to this equation is also 2.80.

We can say that $2m+0.50=6.10$ and $6m+1.50=18.30$ are **equivalent equations** because they have exactly the same solution. Besides 2.80, no other values of m make either equation true. Only the price of $\$2.80$ per pack of markers satisfies the constraint in each purchase.

Equivalent equations: Equations that have exactly the same solutions.

$$2m + 0.50 = 6.10$$

$$6m + 1.50 = 18.30$$

How do we write equivalent equations like these?

There are certain moves we can perform!

In this example, the second equation, $6m + 1.50 = 18.30$, is a result of multiplying each side of the first equation by 3. Buying 3 times as many markers and glue sticks means paying 3 times as much money. The unit price of the markers hasn't changed.

Here are some other equations that are equivalent to $2m + 0.50 = 6.10$, along with the moves that led to these equations.

- $2m + 0.50 = 6.10$
- $2m + 4 = 9.60$ — Add 3.50 to each side of the original equation.

- $2m + 0.50 = 6.10$
- $2m = 5.60$ — Subtract 0.50 from each side of the original equation.

- $2m + 0.50 = 6.10$
- $\frac{1}{2}(2m + 0.50) = 3.05$ — Multiply each side of the original equation by $\frac{1}{2}$.

- $2m + 0.50 = 6.10$
- $2(m + 0.25) = 6.10$ — Apply the distributive property to rewrite the left side.

In each case:

- The move is acceptable because it doesn't change the equality of the two sides of the equation.
- If $2m + 0.50$ has the same value as 6.10, then multiplying $2m + 0.50$ by $\frac{1}{2}$ and multiplying 6.10 by $\frac{1}{2}$ keep the two sides equal.
- Only $m = 2.80$ makes the equation true. Any value of m that makes an equation false also makes the other equivalent equations false. (Try it!)

These moves—applying the distributive property, adding the same amount to both sides, dividing each side by the same number, and so on—might be familiar because we have performed them when solving equations. Solving an equation essentially involves writing a series of equivalent equations that eventually isolates the variable on one side.

Not all moves that we make on an equation would create equivalent equations, however!

For example, if we subtract 0.50 from the left side but add 0.50 to the right side, the result is $2m = 6.60$. The solution to this equation is 3.30, not 2.80. This means that $2m = 6.60$ is *not* equivalent to $2m + 0.50 = 6.10$.

Unit 2 Lesson 5 Practice Problems

1. Which equation is equivalent to the equation $6x + 9 = 12$?

   a. $x + 9 = 6$

   b. $2x + 3 = 4$

   c. $3x + 9 = 6$

   d. $6x + 12 = 9$

2. Select **all** the equations that have the same solution as the equation $3x - 12 = 24$.

   a. $15x - 60 = 120$

   b. $3x = 12$

   c. $3x = 36$

   d. $x - 4 = 8$

   e. $12x - 12 = 24$

3. Jada has a coin jar containing n nickels and d dimes worth a total of $3.65. The equation $0.05n + 0.1d = 3.65$ is one way to represent this situation.

   Which equation is equivalent to the equation $0.05n + 0.1d = 3.65$?

   a. $5n + d = 365$

   b. $0.5n + d = 365$

   c. $5n + 10d = 365$

   d. $0.05d + 0.1n = 365$

4. Select **all** the equations that have the same solution as $2x - 5 = 15$.

   a. $2x = 10$

   b. $2x = 20$

   c. $2(x - 5) = 15$

   d. $2x - 20 = 0$

   e. $4x - 10 = 30$

   f. $15 = 5 - 2x$

5. A basketball coach purchases bananas for the players on his team. The table shows total price in dollars, P, of n bananas.

   Which equation could represent the total price in dollars for n bananas?

Number of bananas	Total price in dollars
7	4.13
8	4.72
9	5.31
10	5.90

   a. $P = 0.59n$

   b. $P = 5.90 - 0.59n$

   c. $P = \frac{5.90}{n}$

   d. $P = n + 0.59$

(From Unit 2, Lesson 3)

6. Kiran is collecting dimes and quarters in a jar. He has collected $10.00 so far and has d dimes and q quarters. The relationship between the numbers of dimes and quarters, and the amount of money in dollars is represented by the equation $0.1d + 0.25q = 10$.

   Select **all** the values (d, q) that could be solutions to the equation.

   a. $(100, 0)$

   b. $(20, 50)$

   c. $(50, 20)$

   d. $(0, 100)$

   e. $(10, 36)$

(From Unit 2, Lesson 4)

7. Bananas cost \$0.50 each, and apples cost \$1.00 each.

   Select **all** the combinations of bananas and apples that Elena could buy for exactly \$3.50.

   a. 2 bananas and 2 apples

   b. 3 bananas and 2 apples

   c. 1 banana and 2 apples

   d. 1 banana and 3 apples

   e. 5 bananas and 2 apples

   f. 5 bananas and 1 apple

   (From Unit 2, Lesson 4)

8. The entrepreneurship club is ordering potted plants for all 36 of its sponsors. One store charges \$8.50 for each plant plus a delivery fee of \$20. The equation $320 = x + 7.50(36)$ represents the cost of ordering potted plants at a second store.

   What does the x represent in this situation?

   a. the cost for each potted plant at the second store

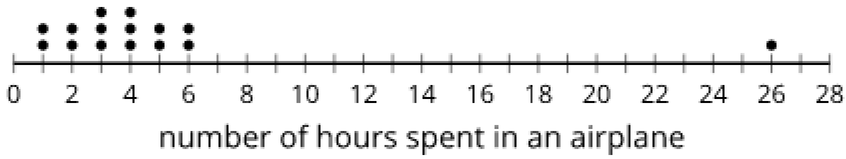
   b. the delivery fee at the second store

   c. the total cost of ordering potted plants at the second store

   d. the number of sponsors of the entrepreneurship club

   (From Unit 2, Lesson 4)

9. The number of hours spent in an airplane on a single flight is recorded on a dot plot. The mean is 5 hours; the median is 4 hours, and the IQR is 3 hours. The value 26 hours is an outlier that should not have been included in the data.



When the outlier is removed from the data set:

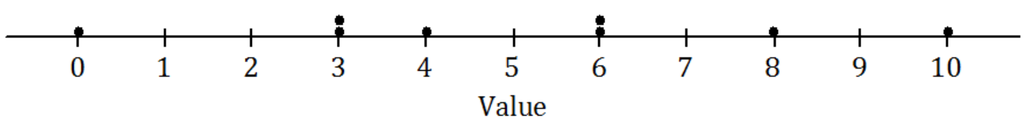
a. What is the mean?

b. What is the median?

c. What is the IQR?

(From Unit 1)

10. Look at the dot plot below.[1]



a. Estimate the mean of this data set.

b. Remember that the standard deviation measures a typical deviation from the mean. The standard deviation of this data set is either 3.2, 6.2, or 9,2. Which of these values is correct for the standard deviation?

(From Unit 1)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

Lesson 6: Explaining Steps for Rewriting Equations

Learning Targets

- I can explain why some algebraic moves create equivalent equations but some do not.
- I know how equivalent equations are related to the steps of solving equations.
- I know what it means for an equation to have no solutions and can recognize such an equation.

Bridge

1. Is $x = 4$ a solution to:

   a. $x(4 + 3) = 28$

   b. $4x + 3x = 28$

2. Is $x = 5$ a solution to:

   a. $x - 4 = -1$

   b. $4 - x = -1$

Warm-up: Could It Be Zero?

Determine if 0 is a solution to each equation.

$4(x + 2) = 10$

$12 - 8x = 3(x + 4)$

$5x = \frac{1}{2}x$

$\frac{6}{x} + 1 = 8$

Activity 1: Explaining Acceptable Moves

Here are some pairs of equations. Partners take turns, row by row, being an explainer and a note taker.

One partner listens and takes notes while the other partner explains why:

- For the equivalent equations, if x is a number that makes the first equation true, then it also makes the second equation true.
- For the non-equivalent equations, the second equation is no longer true for a value of x that makes the first equation true.

Then, switch roles for the following row.

	Equivalent equations	Notes	Non-equivalent equations	Notes
1.	$16 = 4(9 - x)$ $16 = 36 - 4x$		$9x = 5x + 4$ $14x = 4$	
2.	$5x = 24 + 2x$ $3x = 24$		$\frac{1}{2}x - 8 = 9$ $x - 8 = 18$	
3.	$-3(2x + 9) = 12$ $2x + 9 = -4$		$6x - 6 = 3x$ $x - 1 = 3x$	
4.	$5x = 3 - x$ $5x = -x + 3$		$-11(x - 2) = 8$ $x - 2 = 8 + 11$	

Activity 2: It Doesn't Work!

Noah is having trouble solving two equations. In each case, he took steps that he thought were acceptable but ended up with statements that are clearly not true.

Analyze Noah's work on each equation and the moves he made. Were they acceptable moves? Why do you think he ended up with a false equation?

Discuss your observations with your group and be prepared to share your conclusions. If you get stuck, consider solving each equation.

1.

$x+6=4x+1-3x$	original equation
$x+6=4x-3x+1$	apply the commutative property
$x+6=x+1$	combine like terms
$6=1$	subtract x from each side

2.

$2(5+x)-1=3x+9$	original equation
$10+2x-1=3x+9$	apply the distributive property
$2x-1=3x-1$	subtract 10 from each side
$2x=3x$	add 1 to each side
$2=3$	divide each side by x

Lesson Debrief

Lesson 6 Summary and Glossary

When solving an equation, sometimes we end up with a false equation instead of a solution. Let's look at two examples.

Example 1: $4(x+1) = 4x$

Here are two attempts to solve it.

$$\begin{array}{rcll} 4(x+1) & = & 4x & \text{original equation} \\ x+1 & = & x & \text{divide each side by 4} \\ 1 & = & 0 & \text{subtract x from each side} \end{array}$$

$$\begin{array}{rcll} 4(x+1) & = & 4x & \text{original equation} \\ 4x+4 & = & 4x & \text{apply the distributive prop} \\ 4 & = & 0 & \text{subtract 4x from each side} \end{array}$$

Each attempt shows acceptable moves, but the final equation is a false statement. Why is that?

When solving an equation, we usually start by assuming that there is at least one value that makes the equation true. The equation $4(x+1) = 4x$ can be interpreted as: 4 groups of $(x+1)$ are equal to 4 groups of x. There are no values of x that can make this true.

For instance, if $x = 10$, then $x + 1 = 11$. It's not possible that 4 times 11 is equal to 4 times 10. Likewise, 1.5 is 1 more than 0.5, but 4 groups of 1.5 cannot be equal to 4 groups of 0.5.

Because of this, the moves made to solve the equation would not lead to a solution. The equation $4(x+1) = 4$ has no solutions.

Example 2: $2x - 5 = \dfrac{x-20}{4}$

$$\begin{array}{rl} 2x - 5 = \dfrac{x-20}{4} & \text{original equation} \\ 8x - 20 = x - 20 & \text{multiply each side by 4} \\ 8x = x & \text{add 20 to each side} \\ 8 = 1 & \text{divide each side by x} \end{array}$$

Each step in the process seems acceptable, but the last equation is a false statement.

It is not easy to tell from the original equation whether it has a solution, but if we look at the equivalent equation $8x = x$, we can see that 0 could be a solution. When x is 0, the equation is $0 = 0$, which is a true statement. What is going on here?

The last move in the solving process was division by x. Because 0 could be the value of x and dividing by 0 gives an undefined number, we don't usually divide by the variable we're solving for. Doing this might make us miss a solution, namely $x = 0$.

Unit 2 Lesson 6 Practice Problems

1. Match each equation from the left column with an equivalent equation from the right column. Some of the answer choices are not used.

a. $3x + 6 = 4x + 7$	1. $9x = 4x + 7$
b. $3(x + 6) = 4x + 7$	2. $3x + 18 = 4x + 7$
c. $4x + 3x = 7 - 6$	3. $3x = 4x + 7$
	4. $3x - 1 = 4x$
	5. $7x = 1$

2. Mai says that equations A and B have the same solution.

- Equation A: $-3(x + 7) = 24$
- Equation B: $x + 7 = -8$

Which statement explains why this is **true**?

a. Adding 3 to both sides of equation A gives $x + 7 = -8$.

b. Applying the distributive property to equation A gives $x + 7 = -8$.

c. Subtracting 3 from both sides of equation A gives $x + 7 = -8$.

d. Dividing both sides of equation A by -3 gives $x + 7 = -8$.

3. Is 0 a solution to $2x + 10 = 4x + 10$? Explain or show your reasoning.

4. Kiran says that a solution to the equation $x + 4 = 20$ must also be a solution to the equation $5(x + 4) = 100$. Write a convincing explanation as to why this is true.

5. The depth of two lakes is measured at multiple spots. For the first lake, the mean depth is about 45 feet with a standard deviation of 8 feet. For the second lake, the mean depth is about 60 feet with a standard deviation of 27 feet.

   Noah says the second lake is generally deeper than the first lake. Do you agree with Noah?

6. Which equation is equivalent to the equation $5x + 30 = 45$?

   a. $35x = 45$

   b. $5x = 75$

   c. $5(x + 30) = 45$

   d. $5(x + 6) = 45$

(From Unit 2, Lesson 5)

7. Select **all** the equations that have the same solution as $2x-5=15$.

   a. $2x=10$

   b. $2x=20$

   c. $2(x-5)=15$

   d. $2x-20=0$

   e. $4x-10=30$

   f. $15=5-2x$

(From Unit 2, Lesson 5)

8. Diego's age d is 5 more than 2 times his sister's age s. This situation is represented by the equation $d=2s+5$.

   Which equation is equivalent to the equation $d=2s+5$?

   a. $d=2(s+5)$

   b. $d-5=2s$

   c. $d-2=s+5$

   d. $\frac{d}{2}=s+5$

(From Unit 2, Lesson 5)

9. What effect does eliminating the lowest value, -6, from the data set have on the mean and median?

-6, 3, 3, 3, 3, 5, 6, 6, 8, 10

(From Unit 1)

10. For each equation, decide if $x = 12$ is a solution.

a. $6 = \frac{1}{2}x$

b. $18 = 2x$

(Addressing NC.6.EE.5)

Lesson 7: Creating and Solving Equations (Part One)[1]

Learning Targets

- I can solve equations.
- I understand why equations can be solved in multiple ways.

Bridge

Find a solution to each equation:

1. $-3 + x = 8$

2. $-10 = 12 + x$

3. $\frac{2}{3}x = 6$

4. $4x = -26$

Warm-up: Is It a Solution?

For each pair of equations, decide whether the given value of x is a solution to one or both equations:

1. Is $x = 2$ a solution to:
   a. $x(2+3) = 10$
   b. $2x + 3x = 10$
2. $x = 3$ a solution to:
   a. $x - 4 = 1$
   b. $4 - x = 1$
3. $x = 5$ a solution to:
   a. $4x - 5 = 2x + 5$
   b. $4x + 2x = 5 + 5$

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Activity 1: Trading Moves

Your teacher will give you four cards, each with an equation.

1. With your partner, select a card and choose who will take the first turn.
2. During your turn, decide what the next move to solve the equation should be, explain your choice to your partner, and then write it down once you both agree. Switch roles for the next move. This continues until the equation is solved.
3. Choose a second equation to solve in the same way, trading the card back and forth after each move.
4. For the last two equations, choose one each to solve and then trade with your partner when you finish to check each other's work.

Are You Ready for More?

Your class is asked to solve this equation.

$$\frac{1}{2}(x+5)=\frac{1}{3}(2x-4)+8$$

Your friend freezes up a bit when they see fractions. Is there an equivalent equation your friend could use that would get the same solution for x but would eliminate the fractions before distributing?

Lesson Debrief

Lesson 7 Summary and Glossary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.[2]

Suppose we are trying to solve the equation $\frac{4}{5}(x+27)=16$. Two useful ways to start are:

- divide each side by $\frac{4}{5}$
- apply the distributive property

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that $\frac{4}{5}\cdot 27$ will be hard, because 27 isn't divisible by 5. But $16\div\frac{4}{5}$ gives us $16\cdot\frac{5}{4}$, and 16 is divisible by 4. Dividing each side by $\frac{4}{5}$ gives:

$$\begin{aligned} \tfrac{4}{5}(x+27) &= 16 \\ \tfrac{5}{4}\cdot\tfrac{4}{5}(x+27)) &= 16\cdot\tfrac{5}{4} && \text{Multiplying each side of the equation by the same number } (\tfrac{5}{4}) \text{ keeps the two expressions equal} \\ x+27 &= 20 && \text{Simplifying each side of the equation preserves the equality} \\ x &= -7 && \text{Subtracting each side of an equation by the same number preserves the equality} \end{aligned}$$

Sometimes the calculations are simpler if we first use the distributive property. Let's look at the equation $100(x+0.06)=21$. If we first divide each side by 100, we get $\frac{21}{100}$ or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.

$$\begin{aligned} 100(x+0.06) &= 21 \\ 100x+6 &= 21 && \text{Using the distributive property on } 100(x+0.06) \text{ creates an equivalent expression, which must also equal 21} \\ 100x &= 15 && \text{Subtracting the same number from each side of an equation preserves the equality} \\ x &= \tfrac{15}{100} && \text{Dividing each side of an equation by the same number preserves the equality} \end{aligned}$$

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/teachers/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017-2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Unit 2 Lesson 7 Practice Problems

1. Solve each equation and show your work.[3]

   a. $2b + 8 - 5b + 3 = -13 + 8b - 5$

   b. $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

2. Clare was solving an equation, but when she checked her answer she saw her solution was incorrect. It turns out Clare's work has three mistakes!

$$\begin{aligned} 8(2+7x) &= 3x-(2+5x) \\ 10+56x &= 3x-2+5x \\ 10+56x &= 8x-2 \\ 10 &= 64x-2 \\ 12 &= 64x \\ \tfrac{3}{16} &= x \end{aligned}$$

   a. Find all three mistakes.

   b. Find the solution to the equation.

[3] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

3. Consider the equation $3x+4=8x-16$. Solve for x using the given starting point.[4]

Group 1	Group 2	Group 3	Group 4
Subtract $3x$ *from both sides*	*Subtract* 4 *from both sides*	*Subtract* $8x$ *from both sides*	*Add* 16 *to both sides*

4. Determine which of the following equations have the same solution set by recognizing properties rather than solving. Then solve only those that you determine have the same solution set.[5]

a. $2x+3=13-5x$

b. $6+4x=-10+26$

c. $6x+9=\frac{13}{5}-x$

d. $0.6+0.4x=-x+2.6$

e. $3(2x+3)=\frac{13}{5}-x$

f. $4x=-10x+20$

g. $15(2x+3)=13-5x$

h. $15(2x+3)+97=110-5x$

5. The environmental science club is printing T-shirts for its 15 members. The printing company charges a certain amount for each shirt plus a setup fee of $20.

   If the T-shirt order costs a total of $162.50, how much does the company charge for each shirt?

(From Unit 2, Lesson 4)

[4] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[5] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

6. A group of 280 elementary school students and 40 adults are going on a field trip. They are planning to use two different types of buses to get to the destination. The first type of bus holds 50 people, and the second type of bus holds 56 people.

   Andre says that three of the first type of bus and three of the second type of bus will hold all of the students and adults going on the field trip. Is Andre correct? Explain your reasoning.

   (From Unit 2, Lesson 4)

7. Which of the following equations have the same solution set? Give reasons for your answers that do not depend on solving the equations.[6]

   a. $x - 5 = 3x + 7$

   b. $3x - 6 = 7x + 8$

   c. $15x - 9 = 6x + 24$

   d. $6x - 16 = 14x + 12$

   e. $9x + 21 = 3x - 15$

   f. $-0.05 + \frac{x}{100} = \frac{3x}{100} + 0.07$

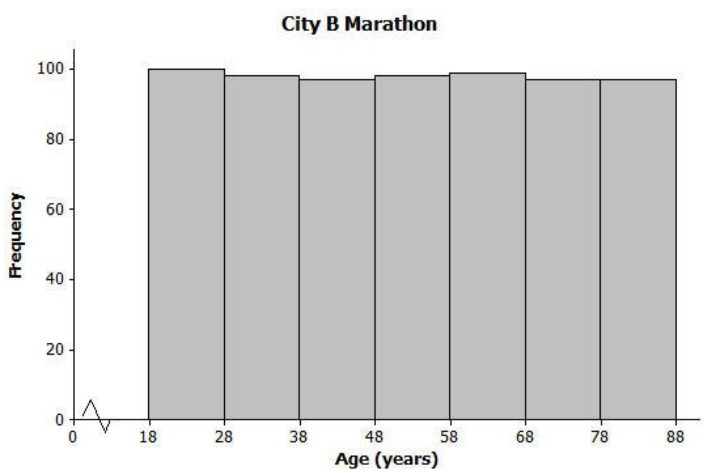
   (From Unit 2, Lesson 6)

[6] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

8. A large city, which we will call City A, holds a marathon. Suppose that the ages of the participants in the marathon that took place in City A were summarized in the histogram below.[7]

a. Make an estimate of the mean age of the participants in the City A marathon.

A smaller city, City B, also held a marathon. However, City B restricts the number of people of each age category who can take part to 100 people. The ages of the participants are summarized in the histogram below.



b. Approximately what was the mean age of the participants in the City B marathon? Approximately what was the standard deviation of the ages?

c. Explain why the standard deviation of the ages in the City B marathon is greater than the standard deviation of the ages for the City A marathon.

(From Unit 1)

[7] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

9. Find the value of each variable.[8]

a. $a \cdot 3 = -30$

b. $-9 \cdot b = 45$

(Addressing NC.7.EE.4)

[8] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 8: Creating and Solving Equations (Part Two)[1]

Learning Targets

- I can solve equations.
- I can solve story problems by writing and solving an equation.

Bridge

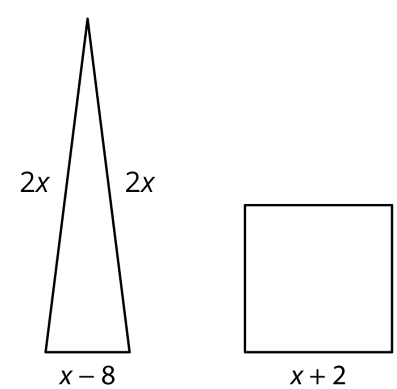
Priya and Elena are both trying to write an expression with fewer terms that is equivalent to $7a + 5b - 3a + 4b$.

- Priya thinks $10a + 1b$ is equivalent to the original expression.
- Elena thinks $4a + 9b$ is equivalent to the original expression.

Who do you agree with and why?[2]

Warm-up: Reflection

The triangle and the square have equal perimeters.



1. Find the value of x.

2. What are the perimeters of each of the figures?

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[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

Activity 1: Squirrels and Chipmunks and More Equations

1. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid three acorns in each of the holes it dug. The squirrel hid four acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed four fewer holes. How many acorns did the chipmunk hide?[3]

2. Mai was hired part time at Shoe Box shoe store as a sales clerk. She earns $84.00 a day plus $3.00 for every pair of shoes she sells. She wants to earn $114 a day. Write an equation and solve to find how many pairs of shoes, *p*, she must sell to make her goal.

3. Coach Hicks and the basketball team bought bottles of water to pass out to those unhoused in their community. The water was on sale for $4.30 per pack of 12. They also needed to pay $3.40 total for reusable bags. They have $55.00 to spend. Write an equation and solve to determine how many packs of water the team can buy.

4. A zookeeper realized that an alligator in the zoo is now four times the age that it was 15 years ago. How old is the alligator now?

[3] Adapted from Illustrativemathematics.org

Are You Ready for More?

The movie was so boring, Andre walked out of the theater after seeing only a quarter of it. Fifteen minutes later, Mae walked out after seeing a third of it. How long was the movie?[4]

Lesson Debrief

[4] Adapted from AchievetheCore.org

Lesson 8 Summary and Glossary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next 6 days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

Before writing an equation, it's important to understand the situation. This might require reading the problem more than once.

Once we understand that this is a problem about running a certain distance each day, we look at the quantities. We see that there is information about the number of miles for both one day (4 miles) and for the total week (22 miles). We see that the same distance was run for each of the 6 days following Monday. We also see that the number of days is a quantity we have to pay attention to. Finally, we want to understand what the question is asking. In this case, the question is, "How many miles did she run on each of the 6 days?" This tells us that the distance Clare runs each of the 6 days, in miles, is unknown. We can represent this quantity by a variable, m.

Since m is the distance Clare runs on each of the 6 days, we can represent the total amount she runs on those days as $6m$. We can then add the 4 miles she runs on Monday to that total, making $4+6m$ the distance Clare ran throughout the week. We also know Clare ran a total of 22 miles throughout the week. This gives the equation $4+6m=22$.

Solving the equation will tell us the distance Clare ran on each of the 6 days.[5]

$$\begin{aligned} 4+6m &= 22 \\ 6m &= 18 \\ m &= 3 \end{aligned}$$

So Clare ran 3 miles on each of the 6 days.

[5] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Unit 2 Lesson 8 Practice Problems

1. Lin and Noah are solving the equation $7(x + 2) = 91$. Lin starts by using the distributive property. Noah starts by dividing each side by 7.

    a. Show what Lin's and Noah's full solution methods might look like.

    b. What is the same and what is different about their methods?[6]

2. A family of six is going to the fair. They have a coupon for $1.50 off each ticket. If they pay $46.50 for all their tickets, how much does a ticket cost without the coupon? Write an equation to find the ticket cost without a coupon.[7]

[6] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

[7] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

3. Here are two stories:[8]

- The initial freshman class at a college is 10% smaller than last year's class. But then during the first week of classes, 20 more students enroll. There are then 830 students in the freshman class.
- A store reduces the price of a computer by \$20. Then during a 10% off sale, a customer pays \$830.

Here are two equations:

- $0.9x + 20 = 830$
- $0.9(x - 20) = 830$

Questions:

a. Decide which equation represents each story.

b. Explain why one equation has parentheses and the other doesn't.

c. Solve each equation and explain what the solution means in the situation.

4. Solve the equations.[9]

a. $2(x-3)=14$

b. $\frac{1}{6}(x+6)=11$

c. $\frac{5}{7}(x-9)=25$

d. $-5(x-1)=40$

5. Write a real-world problem that could be represented by the equation $2(3x+1)=14$ and solve for x.

[9] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

6. Diego was able to verify that $x = 3$ was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been?[10] Identify at least five equations that have a solution of $x = 3$.

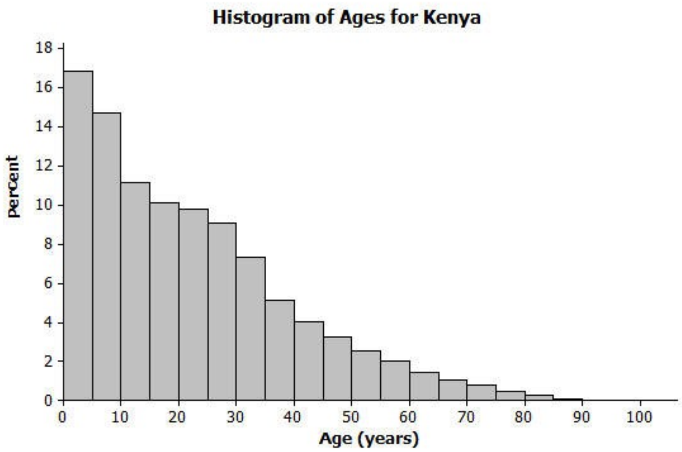
7. Lin ordered 250 pens and 250 pencils to sell for a theatre club fundraiser. The pens cost 11 cents more than the pencils. If Lin's total order costs $42.50, find the cost of each pen and pencil.[11]

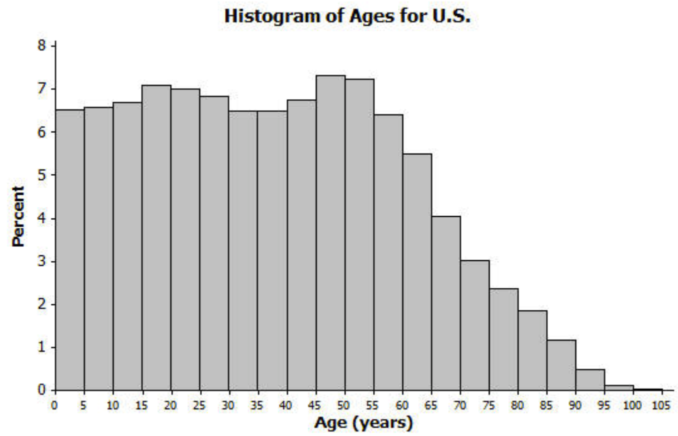
8. Solve the equation below. Identify the moves that are easiest and the moves that are most difficult for you.[12]

$$\frac{2(x+4)}{3} = 12$$

[10] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).
[11] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).
[12] Adapted from Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

9. Review the histograms.[13]





a. Using the histograms of the population distributions of the United States and Kenya in 2010, approximately what percent of the people in the United States were between 15 and 50 years old? Approximately what percent of the people in Kenya were between 15 and 50 years old?

b. What five-year interval of ages represented in the 2010 histogram of the United States age distribution has the most people?

c. Why is the mean age greater than the median age for people in Kenya?

(From Unit 1)

[13] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

Lessons 9 & 10: Checkpoint

Learning Targets

- I can share what I know mathematically.
- I can continue to grow as a mathematician and challenge myself.

Station C: Let's Go to the Carnival

Tyler and Jada go to a carnival during their summer break from school. They have a total of $50.00 to spend.

Charlotte County Carnival	
Admission	Adults 18+ $10.00 Students $7.50
Game tokens	$1.25 each -or- Pay $20 for unlimited tokens
Concessions	Hotdog $4.25 Waffle Fries $3.75 Ice Cream $2.50 Bottled Water $3.00 Soda or Juice $3.25

1. How much money do Tyler and Jada have to spend after they enter the carnival? Write an equation to show how you arrived at your answer.

2. Do Jada and Tyler have enough money to go to the carnival, play five total games, each eat an ice cream and fries, and share one water bottle?

(continued)

Charlotte County Carnival	
Admission	Adults 18+ $10.00 Students $7.50
Game Tokens	$1.25 each -or- Pay $20 for unlimited tokens
Concessions	Hotdog $4.25 Waffle Fries $3.75 Ice Cream $2.50 Bottled Water $3.00 Soda or Juice $3.25

3. Tyler and Jada run into Lin at the carnival. Lin is trying to decide whether to pay for each game token or buy unlimited tokens. Then she realizes it doesn't matter. How many games must she want to play?

4. For lunch, Tyler and Jada bought two hot dogs, shared an order of fries, and ordered some other items. Their total was $18.25. What else did they buy?

5. Your school is planning a field trip to the carnival! Write an equation that could be used to determine the final price, P, for the number of students, s, and the number of chaperones, c, with each student getting $10 for spending money, and three school buses needed for $110 each.

6. The Charlotte-Mecklenburg School rules are that one chaperone is needed for every 10 students. If 125 students are going on the field trip, how much will it cost?

Station D: Weekend Earnings

Prompt: Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay \$12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend \$7.15 on bus fare.

1. Write three or four mathematical questions that could be solved using this information.

2. Write an equation that represents Jada's take-home earnings in dollars, E, if she works at the bookstore for h hours in one day.

3. One day, Jada takes home \$90.45 after working h hours and after paying the bus fare. Write an equation to represent this situation.

4. Is 4 a solution to the last equation you wrote? What about 7?
   - If so, explain how you know one or both of them are solutions.
   - If not, explain why they are not solutions. Then, find the solution.

5. In this situation, what does the solution to the equation tell us? In other words, what does that mean in terms of Jada's story?

6. Jada has a second option to earn money: she could help some neighbors with errands and computer work for $11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend. Which option should she choose—sorting books at the bookstore or helping her neighbors? Explain your reasoning.

7. Jada learned that, according to the Census Bureau data from 2018, women in the United States are paid 82 cents for every dollar earned by men.

    a. Write an equation to represent this statistic.

    b. For the day Jada earned $90.45, how much could she have expected to have earned if she were male?

    c. Jada then learned that the statistics showed that the amount women are paid compared to men in the United States is also influenced by race and age. Research the current statistics on wage earnings for women and men in the United States, based on race and age groups. Choose two subgroups and describe the relationship between their expected earnings.

8. As time permits, answer the questions you created in problem #1.

Station E: Micro-Modeling

1. Diego and his family are planning a trip to Carowinds amusement park next Memorial Day weekend. He wants to ride the Afterburn roller coaster, but as of his 9th birthday, October 26th, he is 52.5 inches tall and the ride has a height requirement of 54 inches. His brother, Kiran, was 51.5 inches on his 9th birthday and 54 inches on his 10th birthday. Is it likely that Diego will be able to ride Afterburn when they go next May? Share your reasoning with words, pictures, and/or numbers.

2. For a rock concert, a rectangular field of size 100 meters by 50 meters was reserved for the audience. The concert was completely sold out, and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert? Justify your response by providing a drawing or explanation.[1]

    a. 2,000

    b. 5,000

    c. 20,000

    d. 50,000

    e. 100,000

[1] Adapted from AchievetheCore.org

Station G: Stacking Cups[2]

In this station, you are tasked with answering the following question:

- How many styrofoam cups would you have to stack to reach the top of your math teacher's head?

Here is some information to get you started:

- My teacher's height is ______ cm.
- A styrofoam cup is approximately _______ cm tall.

You can access the following resources:

- 10 styrofoam cups
- A ruler
- Hints: 5 cards with questions to consider as a small group.

You may not:

- Hold up the cups next to your teacher to "eyeball" the answer

[2] Adapted from Dan Meyer

Lesson 11: Which Variable to Solve For? (Part One)

Learning Targets

- Given an equation, I can solve for a particular variable (like height, time, or length) when the equation would be more useful in that form.
- I know the meaning of the phrase “to solve for a variable.”

Bridge

Use the formula for the area of a rectangle, $A = lw$, to answer the following questions.

As you work, look for patterns or a set of steps that you could use to quickly figure out one measurement, given the others.

1. A rectangle has a length of 4 units and a width of 9 units. Find its area.

2. A rectangle has an area of 50 square units and a width of 10 units. Find its length.

3. A rectangle has an area of 56 square units and a width of 7 units. Find its length.

4. A rectangle has an area of 64 square units and a width of 4 units. Find its length.

5. How would you tell someone to find the measurement of a rectangle's length when given its area and width?

Warm-up: Which Equations?

1. The table shows the relationship between the base length, b, and the area, A, of some parallelograms. All the parallelograms have the same height. Base length is measured in inches, and area is measured in square inches. Complete the table.

b (inches)	A (square inches)
1	3
2	6
3	9
4.5	
$\frac{11}{2}$	
	36
	46.5

2. Decide whether each equation could represent the relationship between b and A. Explain your reasoning.

   a. $b = 3A$

   b. $b = \frac{A}{3}$

   c. $A = \frac{b}{3}$

   d. $A = 3b$

Activity 1: Rewriting Formulas

Equation or formula	What does this formula represent?	Solve for: (Assume no variable is equal to 0)
$d = rt$		r
$d = rt$		t
$y = mx + b$		x
$A = \frac{bh}{2}$		b
$P = 2l + 2w$		w
$E = mc^2$		m
$F = \frac{9}{5}C + 32$		C
$(y - y_1) = m(x - x_1)$		m

Activity 2: Filling and Emptying Tanks

Tank A initially contained 124 liters of water. It is then filled with more water, at a constant rate of 9 liters per minute.

1. How many liters of water are in tank A after the following amounts of time have passed?
    a. 4 minutes
    b. 80 seconds
    c. m minutes
2. How many minutes have passed, m, when tank A contains the following amounts of water?
    a. 151 liters
    b. 191.5 liters
    c. 270.25 liters
    d. p liters

Tank B, which initially contained 80 liters of water, is being drained at a rate of 2.5 liters per minute.

3. How many liters of water remain in the tank after the following amounts of time?

   a. 30 seconds

   b. 7 minutes

   c. t minutes

4. For how many minutes, t, has the water been draining when tank B contains the following amounts of water?

   a. 75 liters

   b. 32.5 liters

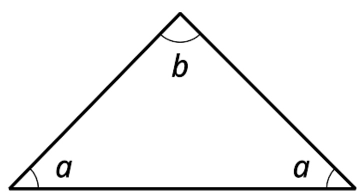
   c. 18 liters

   d. v liters

Lesson Debrief

Lesson 11 Summary and Glossary

A relationship between quantities can be described in more than one way. Some ways are more helpful than others, depending on what we want to find out. Let’s look at the angles of an isosceles triangle, for example.



The two angles near the horizontal side have equal measurement in degrees, a.

The sum of angles in a triangle is $180°$, so the relationship between the angles can be expressed as:

$$a+a+b=180$$

Suppose we want to find a when b is $20°$.

Let's substitute 20 for b and solve the equation.

$$\begin{aligned} a+a+b &= 180 \\ 2a+20 &= 180 \\ 2a &= 180-20 \\ 2a &= 160 \\ a &= 80 \end{aligned}$$

What is the value of a if b is $45°$?

$$\begin{aligned} a+a+b &= 180 \\ 2a+45 &= 180 \\ 2a &= 180-45 \\ 2a &= 135 \\ a &= 67.5 \end{aligned}$$

Now suppose the bottom two angles are $34°$ each. How many degrees is the top angle?

Let's substitute 34 for a and solve the equation.

$$\begin{aligned} a+a+b &= 180 \\ 34+34+b &= 180 \\ 68+b &= 180 \\ b &= 112 \end{aligned}$$

What is the value of b if a is $72.5°$?

$$\begin{aligned} a+a+b &= 180 \\ 72.5+72.5+b &= 180 \\ 145+b &= 180 \\ b &= 35 \end{aligned}$$

Notice that when b is given, we did the same calculation repeatedly to find a: we substituted b into the first equation, subtracted b from 180, and then divided the result by 2.

Instead of taking these steps over and over whenever we know b and want to find a, we can rearrange the equation to isolate a:

$$\begin{aligned} a+a+b &= 180 \\ 2a+b &= 180 \\ 2a &= 180-b \\ a &= \frac{180-b}{2} \end{aligned}$$

This equation is equivalent to the first one. To find a, we can now simply substitute any value of b into this equation and evaluate the expression on the right side.

Likewise, we can write an equivalent equation to make it easier to find b when we know a:

$$\begin{aligned} a+a+b &= 180 \\ 2a+b &= 180 \\ b &= 180-2a \end{aligned}$$

Rearranging an equation to isolate one variable is called “solving for a variable.” In this example, we have solved for a and for b. All three equations are equivalent. Depending on what information we have and what we are interested in, we can choose a particular equation to use.

Unit 2 Lesson 11 Practice Problems

1. Priya is buying raisins and almonds to make trail mix. Almonds cost $5.20 per pound, and raisins cost $2.75 per pound. Priya spent $11.70 buying almonds and raisins. The relationship between pounds of almonds a, pounds of raisins, r, and the total cost is represented by the equation $5.20a + 2.75r = 11.70$.

   How many pounds of raisins did Priya buy if she bought the following amounts of almonds:

   a. 2 pounds of almonds

   b. 1.06 pounds of almonds

   c. 0.64 pounds of almonds

   d. a pounds of almonds

2. Here is a linear equation in two variables: $2x + 4y - 31 = 123$. Solve the equation, first for x and then for y.

3. A chef bought $\$17.01$ worth of ribs and chicken. Ribs cost $\$1.89$ per pound, and chicken costs $\$0.90$ per pound. The equation $0.9c + 1.89r = 17.01$ represents the relationship between the quantities in this situation.

   Show that each of the following equations is equivalent to $0.9c + 1.89r = 17.01$. Then, explain when it might be helpful to write the equation in these forms.

   a. $c = 18.9 - 2.1r$

   b. $r = -\frac{10}{21}c + 9$

4. A car traveled 180 miles at a constant rate.

   a. Complete the table to show the rate at which the car was traveling if it completed the same distance in each number of hours.

Travel time (hours)	Rate of travel (miles per hour)
5	
4.5	
3	
2.25	

   b. Write an equation that would make it easy to find the rate at which the car was traveling in miles per hour, r, if it traveled for t hours.

5. Select **all** the equations that are equivalent to the equation $3x-4=5$.

   a. $3x=9$

   b. $3x-4+4=5+4$

   c. $x-4=2$

   d. $x=9$

   e. $-4=5-3x$

(From Unit 2, Lesson 5)

6. Elena says that equations a and b are not equivalent.

   a. $13-5x=48$

   b. $5x=35$

Write a convincing explanation as to why this is true.

(From Unit 2, Lesson 6)

7. Han is solving an equation. He took steps that are acceptable but ended up with equations that are clearly not true.

$$\begin{array}{rcll} 5x+6 &=& 7x+5-2x & \textit{original equation} \\ 5x+6 &=& 7x-2x+5 & \textit{apply the commutative property} \\ 5x+6 &=& 5x+5 & \textit{combine like terms} \\ 6 &=& 5 & \textit{subtract } 5x \textit{ from each side} \end{array}$$

What can Han conclude as a result of these acceptable steps?

a. There's no value of x that can make the equation $5x+6=7x+5-2x$ true.

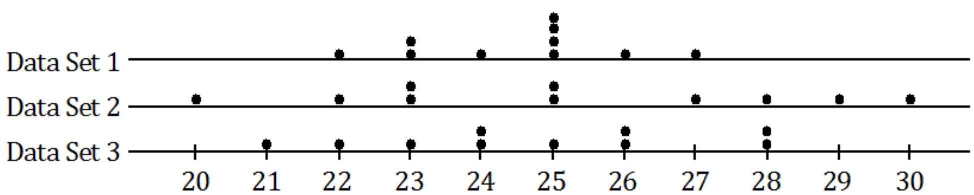
b. Any value of x can make the equation $5x+6=7x+5-2x$ true.

c. $x=6$ is a solution to the equation $5x+6=7x+5-2x$.

d. $x=5$ is a solution to the equation $5x+6=7x+5-2x$.

(From Unit 2, Lesson 6)

8. Three data sets are shown in the dot plots below.[1]



a. Which data set has the smallest standard deviation of the three? Justify your answer.

b. Which data set has the largest standard deviation of the three/? Justify your answer.

(From Unit 1)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

9. Suppose that a teacher plans to give four students a quiz.[2] The minimum possible score on the quiz is 0, and the maximum possible score is 10.

   a. What is the smallest possible standard deviation of the students' scores? Give an example of a possible set of four student scores that would have this standard deviation.

   b. What is the set of four student scores that would make the standard deviation as large as it could possibly be? Use a calculator or Desmos to find this largest possible standard deviation.

(From Unit 1)

10. The formula for the area of a triangle is $A = \frac{1}{2}bh$.

   a. A triangle has a base of 5 units and a height of 4 units. Find its area.

   b. A triangle has an area of 12 square units and a height of 8 units. Find its base.

   c. How would you tell someone to find the length of a triangle's base when given its area and height?

(Building towards NC.M1.A-CED.4)

[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

Lesson 12: Which Variable to Solve For? (Part Two)

Learning Targets

- I can write an equation to describe a situation that involves multiple quantities whose values are not known and then solve the equation for a particular variable.
- I know how solving for a variable can be used to quickly calculate the values of that variable.

Bridge

Sparkling water and grape juice are mixed together to make 36 ounces of fizzy juice.

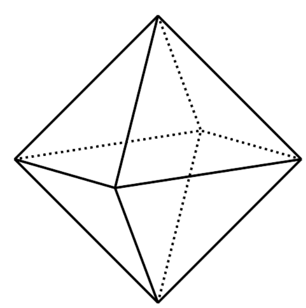
1. How much sparkling water is used if the mixture contains 19 ounces of grape juice?

2. Han wrote the equation $x + y = 36$, with x representing the amount of grape juice used, in ounces, and y representing the amount of sparkling water used, in ounces. Explain why Han's equation matches the story.

Warm-up: Faces, Vertices, and Edges

The equation $V + F - 2 = E$ relates the number of vertices (V), faces (F), and edges (E) in a Platonic solid.

1. Write an equation that makes it easier to find the number of vertices in each of the Platonic solids described:
    - an octahedron (shown here), which has 8 faces
    - an icosahedron, which has 30 edges



2. A Buckminsterfullerene (also called a "Buckyball") is a polyhedron with 60 vertices. It is not a Platonic solid, but the numbers of faces, edges, and vertices are related the same way as those in a Platonic solid.

   Write an equation that makes it easier to find the number of faces a Buckyball has if we know how many edges it has.

Activity 1: Cargo Shipping

An automobile manufacturer is preparing a shipment of cars and trucks on a cargo ship that can carry 21,600 tons.

The cars weigh 3.6 tons each, and the trucks weigh 7.5 tons each.

1. Write an equation that represents the weight limit of a shipment. Let c be the number of cars and t be the number of trucks.

2. For one shipment, trucks are loaded first, and cars are loaded afterwards. (Even though trucks are bulkier than cars, a shipment can consist of all trucks as long as it is within the weight limit.)

   Find the number of cars that can be shipped if the cargo already has:

   a. 480 trucks

   b. 1,500 trucks

   c. 2,736 trucks

   d. t trucks

3. For a different shipment, cars are loaded first, and then trucks are loaded afterwards.

   a. Write an equation you could use to find the number of trucks that can be shipped if the number of cars is known.

   b. Use your equation to find the number of trucks that can be shipped if the cargo already has 1,000 cars. What if the cargo already has 4,250 cars?

Are You Ready for More?

For yet another shipment, the manufacturer is also shipping motorcycles, which weigh 0.3 ton each.

1. Write an equation that you could enter into a calculator or a spreadsheet tool to find the number of motorcycles that can be shipped, m, if the number of cars and trucks are known.

2. Use your equation to find the number of motorcycles that can be shipped if the cargo already contains 1,200 trucks and 3,000 cars.

Lesson Debrief

Lesson 12 Summary and Glossary

Solving for a variable is an efficient way to find out the values that meet the constraints in a situation. Here is an example:

An elevator has a capacity of $3,000$ pounds and is being loaded with boxes of two sizes—small and large. A small box weighs 60 pounds, and a large box weighs 150 pounds.

Let x be the number of small boxes and y the number of large boxes. To represent the combination of small and large boxes that fill the elevator to capacity, we can write:

$$60x + 150y = 3,000$$

If there are 10 large boxes already, how many small boxes can we load onto the elevator so that it fills it to capacity? What if there are 16 large boxes?

In each case, we can substitute 10 or 16 for y and perform acceptable moves to solve the equation. Or, we can first solve for x:

$$\begin{aligned} 60x + 150y &= 3,000 && \textbf{original equation} \\ 60x &= 3,000 - 150y && \textbf{subtract 150y from each side} \\ x &= \tfrac{3,000-150y}{60} && \textbf{divide each side by 60} \end{aligned}$$

This equation allows us to easily find the number of small boxes that can be loaded, x, by substituting any number of large boxes for y.

Now suppose we first load the elevator with small boxes, say, 30 or 42, and want to know how many large boxes can be added for the elevator to reach its capacity.

We can substitute 30 or 42 for x in the original equation and solve it. Or, we can first solve for y:

$$\begin{aligned} 60x + 150y &= 3,000 && \textbf{original equation} \\ 150y &= 3,000 - 60x && \textbf{subtract 60x from each side} \\ y &= \tfrac{3,000-60x}{150} && \textbf{divide each side by 150} \end{aligned}$$

Now, for any value of x, we can quickly find y by evaluating the expression on the right side of the equal sign.

Solving for a variable—before substituting any known values—can make it easier to test different values of one variable and see how they affect the other variable. It can save us the trouble of doing the same calculation over and over.

Unit 2 Lesson 12 Practice Problems

1. A car has a 16-gallon fuel tank. When driven on a highway, it has a gas mileage of 30 miles per gallon. The gas mileage (also called "fuel efficiency") tells us the number of miles the car can travel for a particular amount of fuel (one gallon of gasoline, in this case). After filling the gas tank, the driver got on a highway and drove for a while.

    a. How many miles has the car traveled if it has the following amounts of gas left in the tank?

    - 15 gallons
    - 10 gallons
    - 2.5 gallons

    b. Write an equation that represents the relationship between the distance the car has traveled in miles, d, and the amount of gas left in the tank in gallons, x.

    c. How many gallons are left in the tank when the car has traveled the following distances on the highway?

    - 90 miles
    - 246 miles

    d. Write an equation that makes it easier to find the the amount of gas left in the tank, x, if we know the car has traveled d miles.

2. The area A of a rectangle is represented by the formula $A = lw$ where l is the length and w is the width. The length of the rectangle is 5.

   Write an equation that makes it easy to find the width of the rectangle if we know the area and the length.

3. Noah is helping to collect the entry fees at his school's sports game. Student entry costs $\$2.75$ each, and adult entry costs $\$5.25$ each. At the end of the game, Diego collected $\$281.25$.

   Select **all** equations that could represent the relationship between the number of students, s, the number of adults, a, and the dollar amount received at the game.

   a. $281.25 - 5.25a = 2.75s$

   b. $a = \frac{281.25 - 2.75s}{5.25}$

   c. $281.25 - 5.25s = a$

   d. $281.25 + 2.75a = s$

   e. $281.25 + 5.25s = a$

4. $V = \pi r^2 h$ is an equation to calculate the volume of a cylinder, V, where r represents the radius of the cylinder and h represents its height.

   Which equation allows us to easily find the height of the cylinder because it is solved for h?

   a. $r^2 h = \frac{V}{\pi}$

   b. $h = V - \pi r^2$

   c. $h = \frac{V}{\pi r^2}$

   d. $\pi h = \frac{V}{r^2}$

5. The Department of Streets of a city has a budget of $1,962,800 for resurfacing roads and hiring additional workers this year.

   The cost of resurfacing a mile of two-lane road is estimated at $84,000. The average starting salary of a worker in the department is $36,000 a year.

   a. Write an equation that represents the relationship between the miles of two-lane roads the department could resurface, m, and the number of new workers it could hire, p, if it spends the entire budget.

   b. Take the equation you wrote in the first question and:

      - Solve for p. Explain what the solution represents in this situation.

      - Solve for m. Explain what the solution represents in this situation.

   c. The city is planning to hire six new workers and to use its entire budget.

      - Which equation should be used to find out how many miles of two-lane roads it could resurface? Explain your reasoning.

      - Find the number of miles of two-lane roads the city could resurface if it hires six new workers.

6. A car is going 65 miles per hour down the highway.

   a. How far does it travel in 1.5 hours?

   b. How long does it take the car to travel 130 miles?

   c. Mai wrote the equation $y = 65x$, with x representing the time traveled, in hours, and y representing the distance traveled, in miles. Explain why Mai's equation matches the story.

(From Unit 2, Lesson 3)

7. The table shows the volume of water in cubic meters, V, in a tank after water has been pumped out for a certain number of minutes. Which equation could represent the volume of water in cubic meters after t minutes of water being pumped out?

Time after pumping begins	Volume of water (cubic meters)
0	30
5	27.5
10	20
15	7.5

   a. $V = 30 - 2.5t$

   b. $V = 30 - 0.5t$

   c. $V = 30 - 0.5t^2$

   d. $V = 30 - 0.1t^2$

(From Unit 2, Lesson 4)

8. Which equation has the same solution as $10x – x + 5 = 41$?

   a. $10x + 5 = 41$

   b. $10x – 5 + x = 41$

   c. $9x = 46$

   d. $9x + 5 = 41$

   (From Unit 2, Lesson 5)

9. Noah is solving an equation and one of his moves is unacceptable. Here are the moves he made.

$$\begin{aligned} 2(x+6) &= 8+6x && \textbf{original equation} \\ 2x+12-4 &= 8+6x && \textbf{apply the distributive property} \\ 2x+8 &= 8+6x && \textbf{combine like terms} \\ 2x &= 6x && \textbf{subtract 8 from both sides} \\ 2 &= 6 && \textbf{divide each side by x} \end{aligned}$$

   Which answer **best** explains why the “divide each side by x step” is unacceptable?

   a. When you divide both sides of $2x = 6x$ by x, you get $2x^2 = 6x^2$.

   b. When you divide both sides of $2x = 6x$ by x, it could lead us to think that there is no solution while in fact the solution is $x = 0$.

   c. When you divide both sides of $2x = 6x$ by x, you get $2 = 6x$.

   d. When you divide both sides of $2x = 6x$ by x, it could lead us to think that there is no solution while in fact the solution is $x = 3$.

   (From Unit 2, Lesson 6)

10. This data set represents the number of hours 10 students slept on Sunday night.

6	6	7	7	7	8	8	8	8	9

   Are there any outliers? Explain your reasoning.

   (From Unit 1)

Lessons 13 & 14: Mathematical Modeling[1]

Learning Targets

- I can describe what a modeling prompt is.
- I can explain some elements of a good response to a modeling prompt.

Advice on Modeling

These are some steps that successful modelers often take and questions that they ask themselves. You don't necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.

Understand the Question
Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.

Refine the Question
If necessary, rewrite the question you are trying to answer so that it is more specific.

Estimate a Reasonable Answer
If you don't have enough information to decide what's reasonable, try to come up with an answer that would be too low, and an answer that would be too high.

Identify Unknowns
- What are the meaningful quantities in this situation? Write them down.
- What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.

Gather Information
Write down any of the unknown information that you find. Organize your information in a way that makes sense to you.

Experiment!
Try different ideas to make progress toward answering your question. If you are stuck, think about:
- Helpful ways to organize the information you have or organize your work
- Questions you *can* answer using the information you have
- Ways to represent mathematical relationships or sets of data (tables, equations, graphs, statistical plots)
- Tools that are available for representing mathematics, both digital and analog

Check Your Reasoning
Do you have a first answer to your question? Great! See if it's reasonable.
- Make sure you can explain what the answer means in terms of the original problem.
- Check your precision: Is your answer overly precise? Not precise enough?

Use and Improve Your Model
- Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used?
- What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account.
- What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them?
- What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?

Modeling Rubric

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">1. Decide What to Model</td><td>● Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate.
● Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.</td><td>● Assumptions are noted but lacking in justification or difficult to find.
● Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.</td><td>● No assumptions are stated.
● No variables are defined.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Ask questions about the situation to understand it better
● Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?)
● Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)</td></tr>
<tr><td rowspan="2">2. Formulate a Mathematical Model</td><td>● An appropriate model is chosen and represented clearly.
● Diagrams, graphs, etc. are clear and appropriately labeled.</td><td>Parts of the model are unclear, incomplete, or contain mistakes.</td><td>No model is presented, or the presentation contains significant errors.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Check your model more carefully to make sure it really fits well
● Consider a wider variety of possible models, to find one that fits the situation better
● Think about the situation more deeply before trying to find a model
● Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.</td></tr>
</table>

Skill	Score			Notes or Comments
	Proficient	Developing	Needs Revisiting	
3. Use Your Model to Reach a Conclusion	● Solution is relevant to the original problem. ● Reader can easily understand the reasoning leading to the solution. ● Relevant details are included like units of measure.	Solution is not well-aligned to the original problem, or aspects of the solution are difficult to understand or incomplete.	No solution is provided.	
	To improve at this skill, you could: ● Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later ● Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model ● Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions			
4. Refine and Share Your Model	● The model's implications are clearly stated. ● The limitations of the model and solution are addressed.	The limitations of the model and solution are addressed but lacking in depth or ignoring key components.	No interpretation of model and solution is provided.	
	To improve at this skill, you could: ● Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?" ● Be skeptical of your model: What don't you like about it, and what can you do to fix those things? ● Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.			

Workspace for Modeling Prompt #_____

Modeling Prompt #____ Reflection

Lesson 15: Representing Situations with Inequalities

Learning Target

- I can write inequalities that represent the constraints in a situation.

Warm-up: What Do Those Symbols Mean?

1. Match each inequality to the meaning of the symbol within it.

a. $h > 50$	• less than or equal to
b. $h \leq 20$	• greater than
c. $30 \geq h$	• greater than or equal to

2. Is 25 a solution to any of the inequalities? Which one(s)?

3. Is 40 a solution to any of the inequalities? Which one(s)?

4. Is 30 a solution to any of the inequalities? Which one(s)?

Activity 1: Elevator Constraints

Scenario: An elevator car in a skyscraper can hold at most 15 people. For safety reasons, each car can carry a maximum of 1,500 kg. On average, an adult weighs 70 kg, and a child weighs 35 kg. Assume that each person carries 4 kg of gear with them.

1. With a partner, follow the steps below to complete the table on the next page.
   - Row 1: Use this completed row to guide you in the rows that follow.
   - Row 2: Work together with your partner. Use the equation and variable definitions to explain what constraint the equation represents.
   - Rows 3–4: Work independently. Create two equations or inequalities that you can think of that also represent constraints in the scenario. Define the variables you use and explain how your equation or inequality represents the constraint. (Avoid using the variables z, m, or g.)
   - Rows 5–6:
     - Take turns sharing your equations or inequalities and variable definitions from rows 3 and 4 with one another. Do not share your explanation. Write down your partner's shared information and make sure your partner has correctly written down what you shared. At this point, the only empty boxes in your table should be explanation boxes.
     - Work independently to provide an explanation for the equation or inequality your partner shared with you, using the variable definitions and referring back to the scenario as needed.
     - After both you and your partner have completed the explanations, share the explanations with one another. If there is disagreement, work together to determine whether adjustments need to be made to the equation or inequality to ensure that it is communicated more clearly. If both you and your partner created the same equation or inequality but had different explanations, determine whether the different explanations communicate the same idea.

Row	Example	Equation/Inequality	Explanation	Variables defined
1	Completed example	$p \leq 15$	The elevator car in a skyscraper can hold at most 15 people.	p represents the number of people in an elevator car
2	Partially completed example	$w = 70a + 35c + 4a + 4c$		a represents the number of adults, c represents the number of children in an elevator car, and w represents the weight in kilograms that one elevator car can carry.
3	Your example			
4	Your example			
5	Partner's example			
6	Partner's example			

2. Independently, rewrite the six equations and inequalities (from question 1) so that they would work for a different building where:
   - an elevator car can hold at most z people
   - each car can carry a maximum of m kilograms
   - each person carries g kg of gear

1.	4.
2.	5.
3.	6.

Lesson Debrief

Lesson 15 Summary and Glossary

We have used equations and the equal sign to represent relationships and constraints in various situations. Not all relationships and constraints involve equality, however.

In some situations, one quantity is, or needs to be, greater than or less than another. To describe these situations, we can use inequalities and symbols such as $<, \leq, >$, or $\geq$.

When working with inequalities, it helps to remember what the symbol means, in words. For example:

- $100 < a$ means "100 is less than a."
- $y \leq 55$ means "y is less than or equal to 55," or "y is not more than 55."
- $20 > 18$ means "20 is greater than 18."
- $t \geq 40$ means "t is greater than or equal to 40," or "t is at least 40."

These inequalities are fairly straightforward. Each inequality states the relationship between two numbers ($20 > 18$), or they describe the limit or boundary of a quantity in terms of a number ($100 < a$).

Inequalities can also express relationships or constraints that are more complex. Here are some examples:

- The area of a rectangle, A, with a length of 4 meters and a width of w meters, is no more than 100 square meters.

$$A \leq 100$$
$$4w \leq 100$$

- To cover all the expenses of a musical production each week, the number of weekday tickets sold, d, and the number of weekend tickets sold, e, must be greater than 4,000.

$$d + e > 4,000$$

- Elena would like the number of hours she works in a week, h, to be more than 5 but no more than 20.

$$h > 5$$
$$h \leq 20$$

- The total cost, T, of buying a adult shirts and c child shirts must be less than 150. Adult shirts are $12 each, and children shirts are $7 each.

$$T < 150$$
$$12a + 7c < 150$$

In upcoming lessons, we'll use inequalities to help us solve problems.

Unit 2 Lesson 15 Practice Problems

1. Tyler goes to the store. His budget is $125. Which inequality represents x, the amount in dollars Tyler can spend at the store?

    a. $x \leq 125$

    b. $x \geq 125$

    c. $x > 125$

    d. $x < 125$

2. Jada is making lemonade for a get-together with her friends. She expects a total of 5 to 8 people to be there (including herself). She plans to prepare 2 cups of lemonade for each person.

    The lemonade recipe calls for 4 scoops of lemonade powder for each quart of water. Each quart is equivalent to 4 cups.

    Let n represent the number of people at the get-together, c the number of cups of water, ℓ the number of scoops of lemonade powder.

    Select **all** the mathematical statements that represent the quantities and constraints in the situation.

    a. $5 < n < 8$

    b. $5 \leq n \leq 8$

    c. $c = 2n$

    d. $\ell = c$

    e. $10 < c < 16$

    f. $10 \leq \ell \leq 16$

3. A doctor sees between 7 and 12 patients each day. On Mondays and Tuesdays, the appointments are 15 minutes. On Wednesdays and Thursdays, they are 30 minutes. On Fridays, they are one hour long. The doctor works for no more than 8 hours a day.

    Here are some inequalities that represent this situation.

    $0.25 \leq y \leq 1$

    $7 \leq x \leq 12$

    $xy \leq 8$

    a. What does each variable represent?

    b. What does the expression xy in the last inequality mean in this situation?

4. Han wants to build a dog house. He makes a list of the materials needed:

   - At least 60 square feet of plywood for the surfaces
   - At least 36 feet of wood planks for the frame of the dog house
   - Between 1 and 2 quarts of paint

   Han's budget is $65. Plywood costs $0.70 per square foot; planks of wood cost $0.10 per foot, and paint costs $8 per quart.

   Write inequalities to represent the material constraints and cost constraints in this situation. Be sure to specify what your variables represent.

5. Solve each problem two ways. First, substitute the given values and solve for the given variable. Then, solve for the given variable and substitute for the given values.[1]

   a. The perimeter formula for a rectangle is $p = 2(l + w)$, where p represents the perimeter, l represents the length, and w represents the width. Calculate l when $p = 70$ and $w = 15$.

   b. The area formula for a triangle is $A = \frac{1}{2}bh$, where A represents the area, b represents the length of the base, and h represents the height. Calculate b when $A = 100$ and $h = 20$.

(From Unit 2, Lesson 11)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

6. Elena has $84, and Priya has $12. How much money must Elena give to Priya so that Priya will have three times as much as Elena? Create an equation to represent the problem, and solve for how much Elena must give Priya.[2]

(From Unit 2, Lesson 8)

7. The equation $V = \frac{1}{3}\pi r^2 h$ represents the volume of a cone, where r is the radius of the cone and h is the height of the cone. Which equation is solved for the height of the cone?

    a. $h = V - \pi r^2$

    b. $h = \frac{1}{3}\pi r^2 V$

    c. $3V - \pi r^2 = h$

    d. $h = \frac{3V}{\pi r^2}$

(From Unit 2, Lesson 12)

8. A car is going 65 miles per hour down the highway. Mai wrote the equation $y = 65x$ to match the story, with x representing the time traveled, in hours, and y representing the distance traveled, in miles.

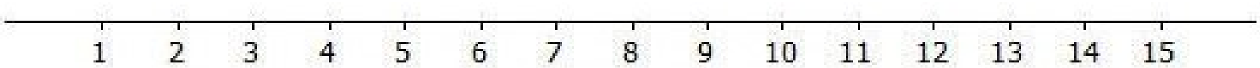
    a. Tyler wrote the equation $x = \frac{y}{65}$, with x representing the time traveled, in hours, and y representing the distance traveled, in miles. Explain why Tyler's equation matches the story.

    b. Lin wrote the equation $y = \frac{x}{65}$, with x representing the time traveled, in hours, and y representing the distance traveled, in miles. Explain why Lin's equation does not match the story.

(From Unit 2, Lesson 12)

9. A data set consisting of the number of hours each of 40 students watched television over the weekend has a minimum value of 3 hours, a Q1 value of 5 hours, a median value of 6 hours, a Q3 value of 9 hours, and a maximum value of 12 hours.[3]

   a. Draw a box plot representing this data distribution.



   b. What is the interquartile range (IQR) for this distribution? What percent of the students fall within this interval?

(From Unit 1)

[3] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Lesson 16: Solutions to Inequalities in One Variable

Learning Targets

- I can graph the solution to an inequality in one variable.
- I can solve one-variable inequalities and interpret the solutions in terms of the situation.
- I understand that the solution to an inequality is a range of values (such as $x > 7$) that makes the inequality true.

Bridge

Solve each inequality for x.

1. $3x + 3 < 12.6$
2. $4x - 2 > 22$

Warm-up: Find a Value, Any Value

1. Write one solution to the inequality $y \leq 9.2$. Explain what makes a value a solution to this inequality.
2. Write one solution to the inequality $7(3 - x) > 14$. Explain your reasoning.

Activity 1: Off to an Orchard

A teacher is choosing between two options for a class field trip to an orchard.

- At orchard A, admission costs $9 per person and 3 chaperones are required.
- At orchard B, the cost is $10 per person, but only 1 chaperone is required.
- At each orchard, the same price applies to both chaperones and students.

1. Determine which orchard would be cheaper to visit for each given number of students:

Number of students	Orchard A	Orchard B
a. 8 students		
b. 12 students		
c. 30 students		

2. To help her compare the cost of her two options, the teacher first writes the equation $9(n+3) = 10(n+1)$ and then she writes the inequality $9(n+3) < 10(n+1)$.

   a. What does n represent in each statement?

   b. In this situation, what does the equation $9(n+3) = 10(n+1)$ mean?

   c. What does the solution to the inequality $9(n+3) < 10(n+1)$ tell us?

   d. Determine the solution set to the inequality $9(n+3) < 10(n+1)$.

Activity 2: Equality and Inequality

1. Solve this equation and check your solution:

   $-\frac{4(x+3)}{5} = 4x - 12$

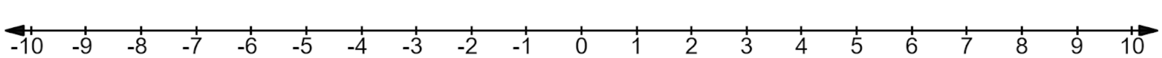
2. Consider the inequality: $-\frac{4(x+3)}{5} \leq 4x - 12$.

   a. Are values less than 2 solutions to the inequality?

   b. Are the values greater than 2 solutions to the inequality?

   c. Choose 2 for x. Is it a solution?

   d. Graph the solution to the inequality on the number line.



Are You Ready for More?

Here is a different type of inequality: $x^2 \leq 4$.

1. Is 1 a solution to the inequality? Is 3 a solution? How about -3?

2. Describe all solutions to this inequality. (If you like, you can graph the solutions on a number line.)

3. Describe all solutions to the inequality $x^2 \geq 9$. Test several numbers to make sure your answer is correct.

Lesson Debrief

Lesson 16 Summary and Glossary

The equation $\frac{1}{2}t = 10$ is an equation in one variable. Its solution is any value of t that makes the equation true. Only $t = 20$ meets that requirement, so 20 is the only solution.

The inequality $\frac{1}{2}t > 10$ is an inequality in one variable. Any value of t that makes the inequality true is a solution. For instance, 30 and 48 are both solutions because substituting these values for t produces true inequalities. $\frac{1}{2}(30) > 10$ is true, as is $\frac{1}{2}(48) > 10$. Because the inequality has a range of values that make it true, we sometimes refer to *all* the solutions as the **solution set**.

Solution set to an inequality: All the values that make the inequality true.

One way to find the solutions to an inequality is by reasoning. For example, to find the solutions to $2p < 8$, we can reason that if 2 times a value is less than 8, then that value must be less than 4. So a solution to $2p < 8$ is any value of p that is less than 4.

Another way to find the solutions to $2p < 8$ is to solve the related equation $2p = 8$. In this case, dividing each side of the equation by 2 gives $p = 4$. This point, where p is 4, is the *boundary* of the solution to the inequality.

To find out the range of values that make the inequality true, we can try values less than and greater than 4 in our inequality and see which ones make a true statement.

Let's try some values less than 4:

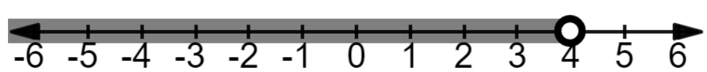
- If $p = 3$, the inequality is $2(3) < 8$ or $6 < 8$, which is true.
- If $p = -1$, the inequality is $2(-1) < 8$ or $-2 < 8$, which is also true.

Let's try values greater than 4:

- If $p = 5$, the inequality is $2(5) < 8$ or $10 < 8$, which is false.
- If $p = 12$, the inequality is $2(12) < 8$ or $24 < 8$, which is also false.

In general, the inequality is false when p is greater than or equal to 4 and true when p is less than 4.

We can represent the solution set to an inequality by writing an inequality, $p < 4$, or by graphing on a number line. The ray pointing to the left represents all values less than 4.



Unit 2 Lesson 16 Practice Problems

1. Here is an inequality: $\frac{7x+6}{2} \leq 3x+2$

   Select **all** of the values that are a solution to the inequality.

   a. $x=-3$

   b. $x=-2$

   c. $x=-1$

   d. $x=0$

   e. $x=1$

   f. $x=2$

   g. $x=3$

2. Find the solution set to this inequality: $2x-3>\frac{2x-5}{2}$

   a. $x<\frac{1}{2}$

   b. $x>\frac{1}{2}$

   c. $x\leq\frac{1}{2}$

   d. $x\geq\frac{1}{2}$

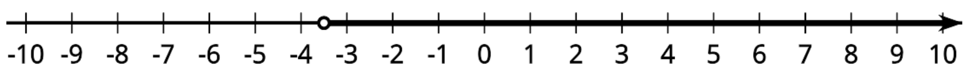
3. Here is an inequality: $\frac{-10+x}{4}+5\geq\frac{7x-5}{3}$

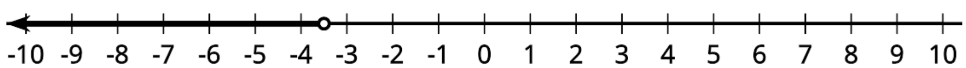
   What value of x will make the two sides equal?

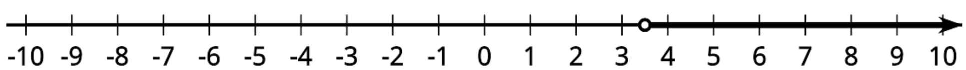
4. Noah is solving the inequality $7x + 5 > 2x + 35$. First, he solves the equation $7x + 5 = 2x + 35$ and gets $x = 6$.

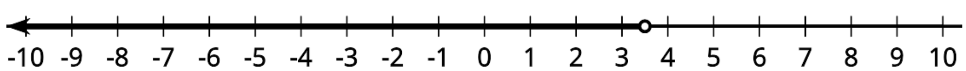
   How does the solution to the equation $7x + 5 = 2x + 35$ help Noah solve the inequality $7x + 5 > 2x + 35$? Explain your reasoning.

5. Which graph represents the solution to $5 + 8x < 3(2x + 4)$?

a. 

b. 

c. 

d. 

6. Solve the following equations:[1]

   a. $-16 - 6v = -2(8v - 7)$

   b. $2(6b + 8) = 4 + 6b$

   c. $7 - 8x = 7(1 + 7x)$

   d. $39 - 8n = -8(3 + 4n) + 3n$

   (From Unit 2, Lesson 8)

7. The principal of a school is hosting a small luncheon for her staff. She plans to prepare two sandwiches for each person. Some staff members offer to bring salads and beverages.

   The principal has a budget of \$225 and expects at least 16 people to attend. Sandwiches cost \$3 each.

   Select **all** of the equations and inequalities that could represent the constraints in the situation, where n is number of people attending and s is the number of sandwiches.

   a. $n \geq 16$

   b. $n \geq 32$

   c. $s < 2n$

   d. $s = 2n$

   e. $3n \leq 225$

   f. $3s \leq 225$

   (From Unit 2, Lesson 15)

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

8. Students at the college are allowed to work on campus no more than 20 hours per week. The jobs that are available pay different rates, starting from $8.75 an hour. Students can earn a maximum of $320 per week.

   Write at least two inequalities that could represent the constraints in this situation. Be sure to specify what your variables represent.

   (From Unit 2, Lesson 15)

9. Two professional race car drivers have the same average lap times after 50 laps. What does it mean to say that the first driver's lap times have a greater standard deviation than the second driver's lap times?

   (From Unit 1)

10. The solution to $5 - 3x > 35$ is either $x > -10$ or $-10 > x$. Which solution is correct? Explain how you know.[2]

    (Addressing NC.7.EE.4)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 17: Writing and Solving Inequalities in One Variable

Learning Targets

- I can analyze the structure of an inequality in one variable to help determine if the solution is greater or less than the solution to the related equation.
- I can write and solve inequalities to answer questions about a situation.

Bridge

Solve each inequality and check your answer using a value that makes your solution true.

1. $-2x < 4$
2. $3x + 5 > 6x - 4$

Warm-up: Dinner for Drama Club

Kiran is getting dinner for his drama club on the evening of their final rehearsal. The budget for dinner is $60.

Kiran plans to buy some prepared food from a supermarket. The prepared food is sold by the pound, at $5.29 per pound. He also plans to buy two large bottles of sparkling water at $2.49 each.

1. Represent the constraints in the situation mathematically. If you use variables, specify what each one means.

2. How many pounds of prepared food can Kiran buy? Explain or show your reasoning.

Activity 1: Different Ways of Solving

Andre and Priya used different strategies to solve the following inequality but reached the same solution.

$$2(2x+1.5)<18-x$$

1. Make sense of each strategy until you can explain what each student has done.

Andre

$$2(2x+1.5)=18-x$$
$$4x+3=18-x$$
$$4x-15=-x$$
$$-15=-5x$$
$$3=x$$

Testing to see if $x=4$ is a solution:

$$2(2\cdot 4+1.5)<18-4$$
$$2(9.5)<14$$
$$19<14$$

The inequality is false, so 4 is not a solution. If a number greater than 3 is not a solution, the solution must be less than 3, or $3>x$.

Priya

$$2(2x+1.5)=18-x$$
$$4x+3=18-x$$
$$5x+3=18$$
$$5x=15$$
$$x=3$$

In $4x+3=18-x$, there is $4x$ on the left and $-x$ on the right.

If x is a negative number, $4x+3$ could be positive or negative, but $18-x$ will always be positive.

For $4x+3<18-x$ to be true, x must include negative numbers or x must be less than 3.

2. Here are four inequalities.

a. $\frac{1}{5}p>-10$

b. $4(x+7)\leq 4(2x+8)$

c. $-9n<36$

d. $\frac{c}{3}<-2(c-7)$

Work with a partner to decide on at least two inequalities to solve. Solve one inequality using Andre's strategy (by testing values on either side of the given solution), while your partner uses Priya's strategy (by reasoning about the parts of the inequality). Switch strategies for the other inequality.

Are You Ready for More?

Using positive integers between 1 and 9, and each positive integer at most once, fill in values to get two constraints so that $x = 7$ is the only integer that will satisfy both constraints at the same time.

$$\square x + \square < \square x + \square$$

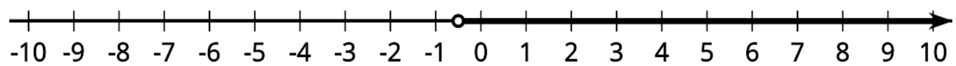
$$\square x + \square > \square x + \square$$

Activity 2: Matching Inequalities and Solutions

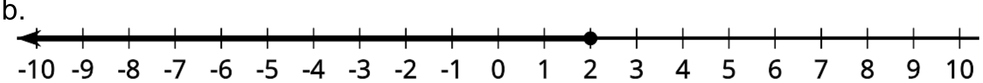
Match each inequality to a graph that represents its solutions. Be prepared to explain or show your reasoning.

1. $5x + 4 \geq 7x$
2. $8x - 2 < -4(x - 1)$
3. $\frac{4x-1}{3} > -1$
4. $\frac{12}{5} - \frac{x}{5} \leq x$

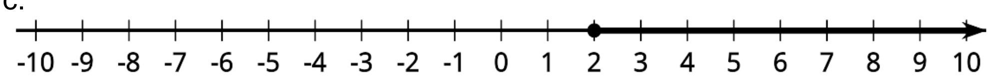
a.



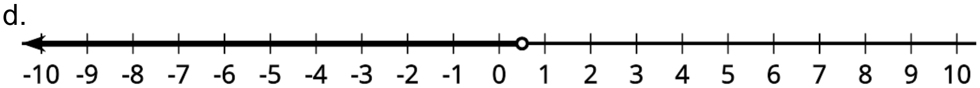
b.



c.



d.



Are You Ready for More?

Diego's goal is to walk more than 70,000 steps this week. The mean number of steps that Diego walked during the first 4 days of this week is 8,019.

a. Write an inequality that expresses the mean number of steps that Diego needs to walk during the last 3 days of this week to walk more than 70,000 steps. Remember to define any variables that you use.

b. If the mean number of steps Diego walks during the last 3 days of the week is 12,642, will Diego reach his goal of walking more that 70,000 steps this week?

Lesson Debrief

Lesson 17 Summary and Glossary

Writing and solving inequalities can help us make sense of the constraints in a situation and solve problems. Let's look at an example.

Clare would like to buy a video game that costs $\$130$. She has saved $\$48$ so far and plans on saving $\$5$ of her allowance each week. How many weeks, w, will it be until she has enough money to buy the game? To represent the constraints, we can write $48+5w \geq 130$. Let's reason about the solutions:

- Because Clare has $\$48$ already and needs to have at least $\$130$ to afford the game, she needs to save at least $\$82$ more.
- If she saves $\$5$ each week, it will take at least $\frac{82}{5}$ weeks to reach $\$82$.
- $\frac{82}{5}$ is 16.4. Any time shorter than 16.4 weeks won't allow her to save enough.
- Assuming she saves $\$5$ at the end of each week (instead of saving smaller amounts throughout a week), it will be at least 17 weeks before she can afford the game.

We can also reason by writing and solving a related equation to find the boundary value for w, and then determine whether the solutions are less than or greater than that value.

$$\begin{aligned} 48+5w &= 130 \\ 5w &= 82 \\ w &= \frac{82}{5} \\ w &= 16.4 \end{aligned}$$

- Substituting 16.4 for w in the original inequality gives a true statement. (When $w=16.4$, we get $130 \geq 130$.)
- Substituting a value greater than 16.4 for w also gives a true statement. (When $w=17$, we get $133 \geq 130$.)
- Substituting a value less than 16.4 for w gives a false statement. (When $w=16$, we get $128 \geq 130$.)
- The solution set is therefore $w \geq 16.4$.

Sometimes the structure of an inequality can help us see whether the solutions are less than or greater than a boundary value. For example, to find the solutions to $3x > 8x$, we can solve the equation $3x = 8x$, which gives us $x=0$. Then, instead of testing values on either side of 0, we could reason as follows about the inequality:

- If x is a positive value, then $3x$ would be less than $8x$.
- For $3x$ to be greater than $8x$, x must include negative values.
- For the solutions to include negative values, they must be less than 0, so the solution set would be $x < 0$.

Unit 2 Lesson 17 Practice Problems

1. Solve $2x < 10$. Explain how to find the solution set.

2. Lin is solving the inequality $15 - x < 14$. She knows the solution to the equation $15 - x = 14$ is $x = 1$.

   How can Lin determine whether $x > 1$ or $x < 1$ is the solution set to the inequality?

3. A cell phone company offers two texting plans. People who use plan A pay 10 cents for each text sent or received. People who use plan B pay 12 dollars per month, and then pay an additional 2 cents for each text sent or received.

   a. Write an inequality to represent the fact that it is cheaper for someone to use plan A than plan B. Use x to represent the number of texts they send.

   b. Solve the inequality.

4. Clare made an error when solving $-4x + 3 < 23$.

   Describe the error that she made.

$$-4x + 3 < 23$$
$$-4x < 20$$
$$x < -5$$

5. Solve $-4+2t-14-18t>-6-100t$, for t in two different ways.[1]

6. Given this kinetic energy formula $K=\frac{p^2}{2m}$,

   a. Solve for m when $K=75$ and $p=10$.

   b. Rearrange the formula to solve for m.

   (From Unit 2, Lesson 11)

7. Solve for x:

   a. $ax+3b=2f$

   b. $rx+h=sx-k$

   c. $3px=2q(r-5x)$

   (From Unit 2, Lesson 11)[2]

---

8. What is the solution set of the inequality $\frac{x+2}{2} \geq -7 - \frac{x}{2}$?

   a. $x \leq -8$

   b. $x \geq -8$

   c. $x \geq -\frac{9}{2}$

   d. $x \geq 8$

(From Unit 2, Lesson 16)

9. Here are statistics for the length of some frog jumps in inches:

   - The mean is 41 inches.
   - The median is 39 inches.
   - The standard deviation is about 9.6 inches.
   - The IQR is 5.5 inches.

   How does each statistic change if the length of the jumps are measured in feet instead of inches?

(From Unit 1)

10. Solve the inequality and check your answer using a value that makes your solution true.

$-3(x+1) \geq 13$

(Addressing NC.7.EE.4)

Lesson 18: Post-Test Activities

Learning Targets

- I understand the reasoning for and will strive to meet the expectations communicated by my teacher.
- I know my classmates and can recognize the value I will add to this classroom community.

Activity 2: Graphs, Tables, Equations, and Situations[1]

1. Take turns with your partner to match a table, a situation, an equation, and a graph. On your turn, you only need to talk about two cards. Eventually all the cards will be sorted into groups of four cards (an equation, situation, table, and graph).

2. For each match that you find, explain to your partner how you know it's a match. Ask your partner if they agree with your thinking.

3. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

[1] Adapted from IM 9–12 Math Algebra 1 Support Course https://curriculum.illustrativemathematics.org/HS/index.html,

Activity 3: Orientation to Graphing Equations with Desmos

1. Look for these features:
    a. On the left: blank rows for listing expressions or equations
    b. On the right: a blank coordinate plane
    c. At the bottom: a keyboard
2. Experiment with these actions:
    a. Figure out how to hide the keyboard and the list of expressions. Then, show both features again.
    b. In a blank row on the left, type $y = 2x + 3$. Notice that a graph appears.
    c. Click the y-intercept of the graph to reveal its coordinates.
    d. Click elsewhere on the graph to see the coordinates of the clicked point.
    e. Drag the point along the graph to see how the coordinates change.
3. Follow these instructions:
    a. Delete the first equation and type $y = 100x + 200$.
    b. Can you see the y-intercept? If not, click the button with a "-" sign (on the right side of the graphing window) to zoom out. Repeat until the y-intercept is visible.
    c. Does it look like the graph overlaps with the vertical axis? If so, click the wrench button in the upper right corner.
    d. Experiment with the scales for the x- and y-axes until the graph seems more useful and the intercepts can be seen more clearly. (For example, these boundaries produce a helpful graphing window: $-10 < x < 10$ and $-50 < y < 250$.)
    e. In a blank row on the left, type $(-1, 100)$. Do you see a point plotted? Click this point to reveal its coordinates, or check the "Label" box that appears in the expression list.